Contracting With Synergies*

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Abstract

This paper studies the effect of synergies on a principal's choice of effort levels and wages for her agents. We model synergies as the extent to which effort by one agent reduces his colleague's marginal cost of effort. It may be optimal to "over-work" and "over-incentivize" a synergistic agent, due to the spillover effects on his colleagues. This effect can rationalize equity grants to rank-and-file employees even if they have little direct effect on productivity, and a high pay differential between CEOs and divisional managers. An agent's pay and effort depend not only on the synergies that he exerts (parameters specific to him) but also the synergies exerted by his colleagues (parameters outside his control). An increase in the synergy between two agents can lead to the third agent being excluded from the team, even if his productivity is unchanged. This result has implications for optimal team composition and firm boundaries.

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1 Introduction

Most work is conducted in teams. In these teams, agents' actions are typically synergistic – effort by one agent reduces the cost of effort for his colleague. For example, going on an international business trip is less costly to a manager if he has an efficient secretary; it is easier for a divisional manager to implement a new workforce practice if the CEO has developed a corporate culture that embraces change. Synergies are also important in non-corporate settings – the cost of giving an academic seminar is lower if one's coauthor has worked hard to improve the quality of the paper.

The structure of synergies within a team is complex. Synergistic relationships are typically asymmetric: a CEO has a greater impact on a divisional manager than the other way round. Moreover, the number of synergistic relationships may vary across agents. A CEO likely exhibits synergies with each of his divisional managers, but a pair of divisional managers might not exhibit synergies with each other.

This paper studies an optimal contracting problem in the presence of such synergies. It analyzes the effect of synergies on optimal pay and effort for not only the synergistic agent but also his colleagues, thus deriving further implications for the effect of synergies on total pay and effort across an organization, and relative pay and effort across agents. In particular, it addresses several questions that cannot be explored in a single-agent framework, such as the determinants of cross-sectional differences in pay across agents, and the optimal composition of a team or boundaries of a firm.

In our theory, agents contribute to the production of a joint output. We model synergies as follows. *Influence* refers to the extent to which effort by one agent reduces the marginal cost of effort of a colleague, and *synergy* is the combination of the unidirectional influences between two agents. Our framework allows for effort to be continuous, influence to be asymmetric across a given pair of agents, and agents to differ in the number of colleagues with whom they enjoy synergies.

We start with a model with N agents and general cost and production functions, to illustrate our influence concept in a broad setting. Without synergies, the effort level that the principal induces from a given agent is a trade-off between the benefits of effort (increased productivity) and its costs (increased wages). With synergies, two additional forces enter the effort determination equation. First, the agent's effort not only has a direct effect on output, but also reduces his colleagues' cost of effort and thus makes it easier to incentivize them. The size of this benefit is represented by the cross-partial derivative of his colleagues' cost function – the extent to which their marginal cost of effort is reduced by an increase in his own effort. Second, if the production function exhibits positive (negative) complementarities, greater effort by the agent increases (decreases) his colleagues' marginal productivity and reduces (increases) the cost of incentivizing them. We study the effect of an increase in one agent's influence – i.e. a more negative cross-partial – on his optimal effort and pay. Even though the direction of the second effect above depends on the production function, we show that an increase in an agent's influence unambiguously raises his optimal effort level, for any production function. In addition, if there are no production complementarities, and if the agent in question is not influenced by any of his colleagues (although they may influence each other), his wage unambiguously increases. The agent is "over-worked" and "over-incentivized" compared to a model without synergy. Giving him a high wage is efficient as his colleagues anticipate that it will induce him to work hard, in turn inducing them to increase effort because of the synergy.

In order to study the effect of changes in an agent's influence on his colleagues' effort and pay – and thus allow analysis of the impact on total effort and pay across all agents, and relative effort and pay between the agents – we specialize the model to the standard setup of a linear production function and quadratic cost function, to achieve tractability. Choosing a particular form for the influence function allows us to represent an agent's influence by a single parameter. Thus, synergy can now be simply represented by the sum of the (unidirectional) influence parameters of a pair of agents. We first consider a two-agent model. Two forces affect the relative effort levels of the agents. It would seem that the more influential agent should work harder, as his effort is more useful because it reduces his colleague's cost function. However, there is a force in the opposite direction: the principal takes advantage of this reduced cost by increasing the effort of his colleague. In our model, these two effects exactly cancel out and both agents exert the same effort level. While this cancellation is specific to our functional forms, two implications are more general. First, an agent's optimal effort level depends not only on his own cost, productivity, and influence, but also positively on his colleague's influence on him. Second, while influence parameters are individual and may be asymmetric across agents, the synergy is common to a pair of agents. It is not only the individual influence parameters, but also the common synergy, that determines the optimal effort level. Indeed, with our functional forms, the common synergy is a sufficient statistic for the optimal effort and the individual influence parameters have no independent effect.

While effort levels are symmetric, wages are not. The more influential agent receives a higher wage upon success. Since the agent is paid zero upon failure, a higher success wage represents both higher incentives and a higher level of expected pay. This asymmetry in the wage occurs even though both agents exert the same effort level (and so pay is not simply a "compensating differential" for disutility), and each agent has the same direct productivity. Instead, higher pay is optimal because it causes the agent to internalize the externalities he exerts on his colleague. When choosing his effort level, each agent takes his colleague's action as given, and so he does not take into account the impact on his colleague's cost of effort. A higher wage causes him to internalize this influence, and so leads him to increase his effort, as desired by the principal.

An increase in the common synergy leads to the principal implementing a higher effort level, and offering higher total wages. The result is consistent with the high level of equity incentives in start-up firms, including to rank-and-file employees. Standard principal-agent theory suggests it is never optimal to give equity to a low-level employees with little direct effect on output. However, particularly in start-up firms where job descriptions are blurred and workers interact frequently with each other, agents can have a significant indirect effect on firm value through aiding their colleagues.

An increase in agent i's influence parameter, holding agent j's influence parameter constant, raises total synergies and so increases total effort and total wages as explained above. Agent i's wage always increases, but the effect on j's wage is more nuanced. It increases if and only if his influence parameter is above a critical threshold, otherwise it decreases. The intuition is as follows. If the principal held j's wage constant, an increase in i's influence would raise j's effort level because it reduces his marginal cost of effort. Thus, the principal could reduce agent j's wage slightly without his effort falling below its previous level. If j's influence is sufficiently low, his effort is less useful to the team than i's. Then, the principal prefers to extract some of the surplus (created by j's lower cost of effort) by lowering agent j's wage, accepting a lower increase in his effort, and reinvesting the savings to further increase i's wage. In contrast, if j's influence is sufficiently high, the principal reinforces the increase in j's effort level by augmenting his wage. In short, synergy determines the (common) effort level and total pay, and influence determines the agents' relative pay.

We then extend the analysis to three agents, which allows us to study differences in the number of synergistic relationships agents enjoy. The "synergy component" refers to the sum of the bilateral influence parameters between a given pair of agents. There are three synergy components, one for each pair of agents. If the synergy components are sufficiently close to each other, all agents exert strictly positive effort, and the ratio of the effort (and thus wage) levels depends on the relative magnitude of all three synergy components. For example, if agent 1 exhibits more synergies with agent 3 than does agent 2, then 1 will exert a higher effort level than 2. Note that the relative effort levels depend on the *total* synergies between each pair of agents, rather than the unidirectional influence parameters. It may seem that whether 1 or 2 exerts higher effort will depend on who exerts more influence on 3, not who is influenced more by 3, since only the former affects the usefulness of their effort. However, if 3 has a greater influence on 1, it is less costly for the principal to induce effort from 1. As in the two-agent model, the optimal effort levels depend on the collective synergy, rather than the individual influence parameters; the latter only affect relative pay.

A natural application of the three-agent model is where one synergy component is close to zero – for example, if two divisional managers exhibit synergies with the CEO but not each other. The two non-synergistic agents can be aggregated into one and the model approximates the two-agent case. Thus, the CEO exerts almost the same effort level as the two divisional managers combined, and so his level of pay is also higher than each divisional manager. Bebchuk, Cremers, and Peyer (2011) interpret a high level of CEO pay compared to other senior managers as inefficient rent extraction, but we show that it can be optimal given the broad scope of a CEO's activities. The increase in CEO in recent decades is consistent with significant developments in communication technologies, which augment a CEO's scope of influence.[1]

If one synergy component becomes sufficiently large compared to the other two, the model collapses to the two-agent setting. Intuitively, if the synergy between two agents is sufficiently strong, then only these two agents matter for the principal – she induces zero effort from the third agent, even though he has the same direct productivity as the first two. Thus, the third agent's participation depends on circumstances outside his control – in contrast to standard models in which an agent's effort level depends only on parameters specific to him (such as his own cost and productivity). Even if his own synergy parameters do not change, if the synergy component between his two colleagues increases, this can lead to him being excluded. This is because the increased synergy between his colleagues raises firm value, and thus the cost of giving the third agent equity to induce effort. This result has interesting implications for the optimal composition of a team – if two agents enjoy sufficiently high synergies with each other, there is no gain in adding a third agent, even if he has just as high direct productivity as the first two. Similarly, if the agents are interpreted as divisions of a firm, the model has implications for firm boundaries. Conventional wisdom suggests

[1] Garicano and Rossi-Hansberg (2006) also show how improvements in communication technologies lead to increased wage inequality within an organization.

that a division should be divested only if it does not exhibit synergies with the rest of the conglomerate. Here, even if a division enjoys strictly positive synergies, it should still be divested if its synergies are lower than those enjoyed by the other divisions – i.e. it is relative, not absolute, synergies that matter.

Our study builds on the literature on multi-agent principal-agent problems. Holmstrom (1982) considers two team-based settings. Where agents contribute to a joint output, a free-rider problem exists. Where each agent has his own output measure, the principal uses relative performance evaluation. There are no synergies in either costs or production.[2] A rich literature, summarized by Bolton and Dewatripont (2005, Chapter 8), has built on both of these settings, analyzing questions such as collusion, mutual monitoring, and the optimal hierarchical structure, but does not consider synergies. Itoh (1991) and Ramakrishnan and Thakor (1991) study a multi-tasking problem where one action increases an agent's own output, and a separate action increases his colleague's output. Here, there is a single team output, and each agent takes a single action which both improves the joint output and reduces his colleague's marginal cost. Some papers have focused on the free-rider problem under production complementarities. Che and Yoo (2001) study a repeated setting, where an agent can punish a shirking colleague by shirking himself in the future. Kremer (1993) studies maximum complementarities in production, when failure in one agent's task leads to automatic failure of the joint project, although agents do not make an effort decision. Winter (2004) extends this framework to incorporate a binary effort choice and shows that it may be optimal to give agents different incentive schemes even if they are ex ante homogenous. Extending this framework further, Winter (2006) studies how the optimal contract depends on the sequencing of agents' actions, and Winter (2010) shows how it depends on the information agents have about each other. Gervais and Goldstein (2007) analyze optimal contracting in a model with production complementarities and agents with self-perception biases. Sakovics and Steiner (2011) study optimal subsidies under production complementarities.

We show that production complementarities are inherently different from cost synergies. In our paper, effort by one agent reduces his colleague's marginal cost of effort. This can also be interpreted as an agent's effort increasing his colleague's marginal private benefit of effort – for example, giving an academic seminar is more enjoyable if one's coauthor has worked hard on the paper. Regardless of the interpretation, the agent does not take into account this externality when making his effort decision. He

[2] In the individual-output model, there is no interaction between the agents; in the joint-output model, the only interaction stems from a joint production function where efforts are perfect substitutes.

is paid according to output; since his synergy does not affect output, his wage must increase to cause him to internalize it. On the other hand, the agent does internalize any increase in production complementarity because this increase also augments his direct productivity. Since the agent is more productive, he will automatically exert the new, higher, effort level even without any change in the contract. In a single-agent model, modifying the production function is isomorphic to modifying the cost function; in a multi-agent model, complementarities in production are fundamentally different from complementarities in costs.

Closest to our paper are other models of contracting with externalities. Kandel and Lazear (1992) study peer pressure, whereby an agent's effort affects the utility of other agents. Their focus is on showing how to model a peer pressure situation, rather than solving for the optimal contract. In Segal (1999), agents exert externalities on each other through their impact on reservation utilities rather than cost functions. The agents' actions are observable participation decisions rather than an unobservable effort choice; there is no output or production function. Studying effort choice (out of a continuum) rather than a zero-one participation decision leads to several new results on the effect of synergies on absolute and relative effort levels. Bernstein and Winter (2012) also focus on a participation decision, as in Segal (1999), and study heterogeneity in externalities. Dessein, Garicano, and Gertner (2010) study the optimal allocation of tasks under economies of scale, which they refer to as synergies. This is a different concept from the synergy in our paper, where effort by one agent reduces another's cost of effort.

While our paper studies the "bright side" that arises when agents interact with each other, there is an extensive literature on the "dark side". Milgrom (1988), Milgrom and Roberts (1988), and Bagwell and Zechner (1993) consider a quite different notion of "influence": wasteful influence costs that agents exert to sway management decisions in their favor, or entrench themselves. Separately, while the pay disparities in our model are an efficient consequence of differences in influence, the literature on tournaments (e.g. Lazear and Rosen (1981)) suggests that they are efficient to incentivize agents.

The paper proceeds as follows. Section 2 illustrates our influence concept in a general model. We specialize the framework to particular functional forms in the two-agent model of Section 3 and the three-agent model of Section 4. Section 5 presents extensions and Section 6 concludes. Appendix A contains proofs and Appendix B studies a variant of the specific model with production complementarities in addition to cost synergies.

2 The Influence Concept

This section outlines our influence concept in a general model. There is a risk-neutral principal ("firm"), and N risk-neutral agents ("workers") indexed $i = 1, 2, \ldots N$. Each agent is protected with limited liability and has a reservation utility of zero. Each agent exerts an unobservable effort level

$$p_i \in [0, 1] \quad i = 1, 2, \ldots N.$$

The agents' efforts affect the firm's output, $r \in \{0, 1\}$, which is publicly observable and contractible. We will sometimes refer to $r = 1$ as "success" and $r = 0$ as "failure". The probability of success depends on the effort levels of all agents as follows:

$$\Pr(r = 1) = P(p_1, \ldots, p_N) = P(p), \tag{1}$$

where $p \equiv (p_1, ..., p_N)$ is a vector containing the effort levels of each agent i. As is standard, we assume that $P_i > 0$ and $P_{ii} < 0 \ \forall \ i$.

The key feature of our model is that each agent's cost of effort $C^i(p)$ depends not only on his own effort level p_i, but also the effort levels exerted by his colleagues. We specify agent i's *overall* cost function as:

$$C^i(p) = k_i g_i(p_i) \Pi_{j \neq i} h_{ji}(p_j) \quad i = 1, 2, \ldots N, \tag{2}$$

The function $g_i(p_i)$ represents agent i's *individual* cost function, and depends on only his own effort level. As is standard, we assume that $g_i'(\cdot) > 0$ and $g_i''(\cdot) > 0$. The function $h_{ji}(p_j)$, where $h_{ji}'(\cdot) < 0$, represents j's *influence* on i, and captures the extent to which effort by j reduces the cost of effort by i. Note that, due to the multiplicative formulation in (2), j's effort reduces not only i's total cost of effort, but also his marginal cost and thus his effort incentives. The parameter k_i is a scaling coefficient which we initially set to unity for all i. The production function P and the cost functions C^i are common knowledge before contracting takes place.

Turning to the contract, it is automatic that each agent i will be paid zero upon failure. The principal chooses the optimal wage $w_i \geq 0$ to pay agent i upon success, that maximizes her expected output net of wages. After wages have been chosen, each agent i, observing the entire wage profile, selects his effort p_i to maximize his expected utility, given by his wage minus his cost of effort. Agents choose their efforts simultaneously, and their effort levels constitute a Nash Equilibrium.

Before we move to the analysis, a couple of points about the setup are worth making. First, agent i sets his effort p_i without observing his colleagues' effort levels (but rather only correctly expecting them in equilibrium). Since i's cost of effort depends on his colleagues' effort levels, this implies that agent i chooses his own effort without observing the implied cost (only correctly expecting it in equilibrium). The model also applies to the case in which agents choose their efforts sequentially, but each agent does not observe the efforts already taken by other agents when choosing his own. Either simultaneous efforts, or sequential efforts without knowing the effort of one's predecessors, apply to many applications. For example, a CEO may commit to a business trip and exert effort in advance to make it successful, but the exact cost he bears in making the trip will depend on the preparation conducted by his secretary, which is not known to the CEO until after the trip is underway. Alternatively, the cost function C^i may contain elements of private benefit – e.g., the extent to which the CEO enjoys her trip – which again depend on the efforts of her colleagues.

Second, since the agent is paid zero upon failure (which is a consequence of risk neutrality, limited liability, and zero reservation utility), an increase in w_i corresponds to an increase in both incentives (the sensitivity of pay) and expected pay (the level of pay, which is often referred to as the "wage" in empirical studies). Thus, in the analysis that follows, all results pertaining to w_i are predictions for both the level and sensitivity of pay: an increase (decrease) in w_i raises (reduces) both. These predictions do not hinge upon risk neutrality but will continue to hold in a model with risk aversion and a binding participation constraint. An increase in the sensitivity of pay will cause the agent to demand a risk premium, augmenting the level of pay.

2.1 Analysis

The principal solves for the optimal vector of effort levels, $p^* = (p_1^*, ..., p_N^*)$, that solves:

$$\max_{\{p_i\},\{w_i\}} P\left(p_1, \ldots, p_N\right) \left(1 - \sum_i w_i\right) = PM, \tag{3}$$

subject to the N incentive compatibility (IC) conditions for each agent i:

$$p_i \in \arg\max_{p_i} w_i P(p) - C^i\left(p\right) \qquad i = 1, 2, \ldots N. \tag{4}$$

Assuming that the first-order approach is valid, we can replace each incentive compatibility constraint (4) by the corresponding first-order condition:

$$w_i P_i(p) - C_i^i(p) = 0, \tag{5}$$

which yields the optimal wage as

$$w_i = \frac{C_i^i}{P_i}.$$

Differentiating the principal's objective function, p_i^* satisfies the first-order condition:

$$\frac{\partial}{\partial p_i}|_{p^*} PM = f_i(p^*) = 0 = P_i\left[1 - \sum_i w_i\right] + P\left[-\frac{C_{ii}^i P_i - C_i^i P_{ii}}{P_i^2} - \sum_{j\neq i}\frac{C_{ij}^j P_j - C_j^j P_{ij}}{P_j^2}\right] \tag{6}$$

Substituting for C^i, (6) can be written as

$$\begin{aligned} 0 =& P_i\left[1 - \sum_i w_i\right] \\ & - P\frac{g_i''(p_i^*)\Pi_{j\neq i}h_{ji}(p_j^*)P_i - g_i'(p_i^*)\Pi_{j\neq i}h_{ji}(p_j^*)P_{ii}}{P_i^2} \\ & - P\sum_{j\neq i} g_j'\left(p_j^*\right)\frac{h_{ij}'(p_i^*)\Pi_{m\neq i,j}h_{mj}(p_m^*)P_j - \Pi_{j\neq i}h_{ij}(p_i^*)P_{ij}}{P_j^2} \end{aligned} \tag{7}$$

The first-order condition with respect to p_i, (7), contains three terms. The first is the increase in firm value from augmenting p_i, due to increased production, multiplied by the principal's share after paying wages to all agents. The second is the increased wage that the principal needs to pay agent i to induce this higher effort level, which results from the convexity of the cost function ($g_i'' > 0$) and the concavity of the production function ($P_{ii} < 0$). These two terms are standard in models without synergy and the optimal effort level is a trade-off between them.

The third term is specific to a model of synergy and is the key focus of this paper. The first term in the numerator arises because an increase in i's effort reduces the cost of effort of all of his colleagues j, since $h_{ij}'(p_i^*) < 0$. This in turn reduces the wage that the principal must pay to j, and represents a benefit to the principal. This force tends to induce the principal to implement a higher p_i^* than in the absence of synergies. The second term arises because an increase in i's effort changes the marginal productivity of j's effort, due to production synergies, which also changes the wage the principal must pay to agent j. If $P_{ij} > 0$ (production complementarities are positive), this represents a benefit to the principal: effort by i increases j's marginal productivity, so a lower wage is required. If $P_{ij} < 0$ (production complementarities are negative), the opposite is the case. In sum, the third term in the first-order condition (7) demonstrates two additional forces that the principal takes into account when determining effort levels in a model with synergies.

We now consider the effect of an increase in agent 1's influence on his optimal effort p_1^* and optimal wage w_1^*. Suppose that agent 1's influence on the other agents j, $j \neq 1$, (weakly) increases so that the function $h_{1j}(p_j)$ shifts to $\tilde{h}_{1j}(p_j)$, where $\tilde{h}_{1j} \leq h_{1j}$ and $\tilde{h}'_{1j} \leq h'_{1j}\ \forall\ j \neq 1$, with $\tilde{h}_{1j} < h_{1j}$ and $\tilde{h}'_{1j} < h'_{1j}$ for at least one j. Such a shift has two effects. First, it reduces agent j's cost function C^j and marginal cost C^j_j, which now become:

$$C^j(p) = g_j(p_j)\,\widetilde{h}_{1j}(p_1)\prod_{m\neq 1,j} h_{mj}(p_m)$$
$$C^j_j(p) = g'_j(p_j)\,\widetilde{h}_{1j}(p_1)\prod_{m\neq 1,j} h_{mj}(p_m).$$

Such a reduction would occur even if 1's effort level were held constant at p_1^*. This effect could be similarly achieved by reducing j's cost coefficient k_j, and thus it could be studied in a model without synergies. Second, it reduces the cross-partial C^j_{1j}: the magnitude of the reduction in j's marginal cost C^j_j depends on 1's effort level. In particular, the cross-partial C^j_{1j} is given by:

$$C^j_{1j}(p) = g'_j(p_j)\,\widetilde{h}'_{1j}(p_1)\prod_{m\neq 1,j} h_{mj}(p_m) < 0.$$

The cross-partial is negative since $\widetilde{h}'_{1j}(p_1) < 0$. This second effect is the focus of our model: the cross-partial is specific to a model of synergy and cannot be achieved by simply changing the coefficient k_j. To isolate this second effect, we shift agent 1's influence function in a way that reduces the cross-partial C^j_{1j}, but keeps C^j and C^j_j unchanged at 1's old optimal effort level. We do so by setting $k_j = \frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)}$, so that j's new cost function is:

$$\widetilde{C}^j(p) = \frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)} g_j(p_j)\tilde{h}_{1j}(p_1)\prod_{m\neq 1,j} h_{mj}(p_m). \tag{8}$$

We can immediately see that $C^j(p_1^*, p_2, ..., p_N) = \widetilde{C}^j(p_1^*, p_2, ..., p_N)$: if p_1 is held constant at p_1^*, j's cost function is unchanged.

The principal's objective function (3) becomes

$$P\left(1-\frac{C_1^1}{P_1}-\sum_{j\neq 1}\frac{\widetilde{C}_j^j}{P_j}\right)=P\widetilde{M}.$$

Let the solution to $P\widetilde{M}$ be $\tilde{p}^*$. By the definition of k_j we have, for all $j\neq 1$:

$$\begin{aligned}\widetilde{C}_j^j(p^*)&=\frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)}g_j'(p_j^*)\tilde{h}_{1j}(p_1^*)\prod_{m\neq 1}h_{mj}\left(p_m^*\right)\\&=g_j'(p_j^*)h_{1j}(p_1^*)\prod_{m\neq 1}h_{mj}\left(p_m^*\right)=C_j^j(p^*),\end{aligned}$$

and for all $j\neq 1$, $i\neq 1$:

$$\begin{aligned}\widetilde{C}_{ji}^j(p^*)&=\frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)}g_j'(p_j^*)\tilde{h}_{1j}(p_1^*)h_{ji}'(p_i^*)\prod_{m\neq 1,i}h_{mj}(p_m^*)C_i^i(p^*)\\&=g_j'(p_j^*)h_{1j}(p_1^*)h_{mj}'(p_m^*)\prod_{m\neq 1,i}h_{mj}(p_m^*)C_i^i(p^*)=C_{ji}^j(p^*).\end{aligned}$$

Therefore, for all $j\neq 1$,

$$\begin{aligned}\frac{\partial}{\partial p_j}\mid_{p^*}P\widetilde{M}&=P_j\left[1-\frac{C_1^1}{P_1}-\sum_{i\neq 1}\frac{\widetilde{C}_i^i}{P_i}\right]+P\left[-\frac{C_{1j}^1P_1-C_1^1P_{1j}}{P_1^2}-\sum_{i\neq 1}\frac{\widetilde{C}_{ij}^iP_i-\widetilde{C}_i^iP_{ij}}{P_i^2}\right]\\&=P_j\left[1-\sum_i\frac{C_i^i}{P_i}\right]+P\left[-\sum_i\frac{C_{ij}^iP_i-C_i^iP_{ij}}{P_i^2}\right]\\&=\frac{\partial}{\partial p_j}|_{\{p_i^*\}}PM=0.\end{aligned}\tag{9}$$

It is intuitive that the partial derivative of p_j, $j\neq 1$, remains zero under the new cost function if all effort levels are held constant. The increase in 1's influence only affects the cross-partials, and thus only affects the first-order conditions if p_1 changes. Thus, if p_1 remains constant at p_1^*, the first-order conditions are unchanged.

We now turn to agent 1. We have:

$$\begin{aligned}
\frac{\partial}{\partial p_1} \mid_{p^*} P\widetilde{M} &= P_1 \left[1 - \frac{C_1^1}{P_1} - \sum_{j \neq 1} \frac{\widetilde{C}_j^j}{P_j}\right] + P \left[-\frac{C_{11}^1 P_i - C_1^1 P_{11}}{P_1^2} - \sum_{j \neq 1} \frac{\widetilde{C}_{j1}^j P_j - \widetilde{C}_j^j P_{j1}}{P_j^2}\right] \\
&= \frac{\partial}{\partial p_1} \mid_{p^*} PM + \sum_{j \neq 1} \frac{P}{P_j} \left(C_{j1}^j - \widetilde{C}_{j1}^j\right) \\
&= \sum_{j \neq 1} \frac{P}{P_j} \prod_{i \neq 1} h_{ij}(p_i^*) g_j'(p_j^*) \left[h_{1j}'(p_1^*) - k_j \tilde{h}_{1j}'(p_1^*)\right]. \qquad (10)
\end{aligned}$$

where the last line follows because $\frac{\partial}{\partial p_1} \mid_{p^*} PM = 0$, since p^* solved the original maximization problem PM. Notice that the P_{j1} terms disappear. In the first-order condition (7) we noted that introducing synergies creates two additional effects in the principal's choice of 1's effort level: effort by agent 1 reduces j's marginal cost of effort and changes his marginal benefit of effort. The sign of the second effect was ambiguous as it depended on whether the production function exhibits positive or negative complementarities. Thus, it may seem that the cross-partial term P_{j1} would be critical in determining how p_1^* changes under the new influence function. However, these production complementarities disappear, as they were already taken care of in the original problem PM. Surprisingly, the effect on p_1^* does not depend on the production function. The intuition is that we are only changing agent 1's influence function, and keeping the production function constant. Thus, whatever effect the production function has on the new principal's problem, maximizing $P\widetilde{M}$, was already present under the old problem PM, and so does not change the effect of varying 1's influence function on the optimal effort levels.

Since $\tilde{h}_{1j}' \leq h_{1j}' < 0$ and $k_j \geq 1 \ \forall \ j \neq 1$, with $\tilde{h}_{1j} < h_{1j}$ and $k_j > 1$ for at least one j, we have:

$$\frac{\partial}{\partial p_1} \mid_{p^*} P\widetilde{M} > 0. \qquad (11)$$

By (11), there exists some p' where $p_1' > p_1^*$ such that $P\widetilde{M} \mid_{p'} > P\widetilde{M} \mid_{p^*}$. By optimality $P\widetilde{M} \mid_{\tilde{p}^*} > P\widetilde{M} \mid_{p^*}$. Suppose $\tilde{p}_1^* \leq p_1^*$. Then by concavity, there exists some $p = (p_1^*, \hat{p}_2, ..., \hat{p}_N)$ such that $P\widetilde{M} \mid_p > P\widetilde{M} \mid_{p^*}$. However, this contradicts (9), which says that given p_1^*, $(p_2^*, ..., p_N^*)$ optimizes $P\widetilde{M}$. Thus, we have $\tilde{p}_1^* > p_1^*$.

We now study the effect on agent 1's wage w_1^*. We have

$$\begin{aligned} w_1^* &= \frac{C_1^1(p_1^*)}{P_1(p^*)} = \frac{g_1\left(p_1^*\right)\Pi_{j\neq i}h_{ji}\left(p_j\right)}{P_1\left(p^*\right)} \\ \tilde{w}_1^* &= \frac{C_1^{1\prime}(\widetilde{p}_1^*)}{P_1(\widetilde{p}^*)} = \frac{g_1'\left(\widetilde{p}_1^*\right)\Pi_{j\neq i}h_{ji}\left(\widetilde{p}_j\right)}{P_1\left(\widetilde{p}^*\right)}. \end{aligned}$$

We have $g_1'\left(\widetilde{p}_1^*\right) > g_1'\left(p_1^*\right)$, since $\widetilde{p}_1^* > p_1^*$ and costs are convex. In addition, we have $P_1\left(\widetilde{p}_1^*, \cdot\right) < P_1\left(p_1^*, \cdot\right)$, since $\widetilde{p}_1^* > p_1^*$ and the production function is concave. Both effects tend to increase w_1^*. However, it is not automatic that $\tilde{w}_1^* > w_1^*$ due to two potentially confounding effects. First, even though $P_1\left(\widetilde{p}_1^*, \cdot\right) < P_1\left(p_1^*, \cdot\right)$, we cannot conclude that P_1 has decreased overall, since it depends also on $(p_2^*, ..., p_N^*)$ which has changed to $(\widetilde{p}_2^*, ..., \widetilde{p}_N^*)$.[3] If p_j^* has risen and $P_{1j} > 0$, or p_j^* has fallen and $P_{1j} < 0$, then this tends to increase 1's marginal productivity and decrease w_1^*. Second, if p_j^* has risen, then $h_{j1}\left(\widetilde{p}_j^*\right) < h_{j1}\left(p_j^*\right)$, which also tends to decrease w_1. These two effects disappear, allowing us to show unambiguously that $\tilde{w}_1^* > w_1^*$, in a useful benchmark case. If there are no complementarities in production, then $P_{1j} = 0 \; \forall \; j$ and so $P_1\left(\widetilde{p}^*\right) < P_1\left(p^*\right)$. If agent 1 is not influenced by his colleagues, then $h_{j1}\left(\cdot\right) = 1 \; \forall \; j$. Note that $h_{j1}\left(\cdot\right) = 1$ still allows the other $N - 1$ agents to influence each other.

The economics behind the increase in wage are as follows. Agent 1 is now more influential, and so the principal wishes to induce greater effort from him. However, 1's greater influence does not give him incentives to exert this greater effort level. He is paid according to output, and so his incentives to exert effort depend on the effect of his effort on output. Changing his influence function $h_{1j}\left(\cdot\right)$ has no effect on his direct productivity, but instead only affects output indirectly through changing j's cost of effort and in turn affecting his effort choice. In a Nash equilibrium, when choosing his effort level, 1 takes j's effort choice as given and so he does not internalize his externality. Holding the other agents' efforts constant, 1's marginal benefit from exerting effort is $w_1 P_1$ and unaffected by the change in his influence function. Thus, the principal increases w_1 to augment agent 1's marginal benefit, and cause him to internalize this externality. Agent 1 is thus "over-incentivized" compared to a model without synergy. Importantly, this result illustrates the difference between our approach of modeling the complementarity between the agents in the cost function (or private benefit function), and an alternative approach of modeling it in the production function. Under the

[3] Whether $\widetilde{p}_j^* > p_j^*$ for $j \neq 1$ depends on the functional forms for the cost and production functions. In Sections 3 and 4 we consider specific functional forms which allows a full explicit analysis of the effect on p_j^*.

alternative approach, the complementarity would affect the agent's marginal productivity and would be internalized even under the original contract. Thus, wages may be independent of the complementarity. We will return to this point in Section 5.2.

The results in this section are summarized in Theorem 1 below.

Theorem 1 *Consider a production function P and a set of N cost functions $C^i(p)$. Let p^*, w^* denote the optimal effort and wage vectors chosen by the principal. Consider a change in agent j's cost function to $\widetilde{C}^j(p) = \frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)} g_j(p_j)\tilde{h}_{1j}(p_1)\prod_{i\neq 1} h_{ij}(p_i)$, where $\tilde{h}_{1j} \leq h_{1j}$ and $\tilde{h}'_{1j} \leq h'_{1j}$ $\forall$ $j \neq 1$, with $\tilde{h}_{1j} < h_{1j}$ and $\tilde{h}'_{1j} < h'_{1j}$ for at least one j. Let $\widetilde{p}^*, \widetilde{w}^*$ denote the new optimal effort and wage vectors chosen by the principal. We have*

(i) $\widetilde{p}_1^* > p_1^*$

(ii) If $P_{1j} = 0$ and $h_{j1}(\cdot) = 1$ $\forall$ j, $\widetilde{w}_1^* > w_1^*$.

While the above analysis studies the effect of changing 1's influence function on his own effort and wage levels, we are also interested in the impact on his colleagues' effort and wage levels. Not only are these of independent interest, but solving for them also allows us to study the relative effort and wage levels between the agents, as well as the total effort and wage levels across the organization. The effect on 1's colleagues cannot be studied tractably in the current setup, since the production and cost functions are very general. Thus, to proceed further and generate additional results, we now select specific functional forms for the production and cost functions. An additional advantage of choosing functional forms is that we can now capture each agent's influence by a parameter h_{ij}, rather than a function $h_{ij}(\cdot)$. This allows the relative influence levels of each agent to be easily compared, and also the influence parameters to be aggregated to measure the combined synergy between a pair of agents. We start with a two-agent model, as this illustrates the additional results most clearly, and then move to a three-agent model which delivers further implications.

3 The Two-Agent Model

We now specialize the production function (1) to the following:

$$\Pr(r = 1) = \frac{p_1 + p_2}{2}. \tag{12}$$

We choose a production function with perfect substitutes to maximize transparency: since there are no complementarities in the production function, we can be sure that it is complementarities in the cost function (i.e. synergies) that are driving the results. Appendix B studies a similar model in which efforts are perfect complements, and shows that the main results continue to hold. Also for transparency, we specify the two agents as having the same direct productivity on output (and, as we show below, the same individual cost function). Thus, any differences in effort and wages are a result of differences in influence, rather than in direct productivity or individual cost.

We specify a quadratic individual cost function:

$$g_i(p_i) = \frac{1}{4}p_i^2. \tag{13}$$

The influence function is given by:

$$h_{ji}\left(p_j\right) = 1 - h_{ji}p_j,$$

where

$$\begin{aligned} h_{ji} &\geq 0 \quad 1 \leq i \neq j \leq N \\ \forall i,\ \sum_{j\neq i} h_{ji} &< 1. \end{aligned}$$

The variable h_{ji} is an *influence parameter* that represents the extent to which i's effort reduces j's cost of effort. For now we consider the case of non-negative influence parameters; in Section 5.1 we allow for $h_{ji} < 0$.

We set the scaling parameter k_i to unity throughout the analysis.[4] Agent i's overall cost function is thus:

$$C^i\left(p\right) = \frac{1}{4}p_i^2\left(1 - h_{ji}p_j\right). \tag{14}$$

The principal's program now specializes to:

$$\max_{\{p_i\},\{w_i\}} \frac{p_1 + p_2}{2}\left(1 - w_1 - w_2\right), \tag{15}$$

subject to the incentive compatibility (IC) conditions for each agent i:

$$p_i \in \arg\max_{p_i} \frac{p_i + p_j}{2} w_i - \frac{1}{4}p_i^2\left(1 - h_{ji}p_j\right) \qquad i = 1, 2. \tag{16}$$

[4]In the model of Section 2, we needed to change k when changing the influence functions in order to permit a tractable analysis. In this section, given the tractability of the model, a clean analysis is possible without any rescaling.

Differentiating i's utility function (16) gives his first-order condition as:

$$w_i = p_i(1 - h_{ji}p_j). \tag{17}$$

Plugging this into the principal's objective function (15) gives her reduced-form maximization problem as:

$$p_1^*, p_2^* \in \arg\max_{p_1,p_2} \frac{p_1 + p_2}{2} \left(1 - (p_1 + p_2) + p_1 p_2 (h_{12} + h_{21})\right). \tag{18}$$

We define the following term:

Definition 1 ***Synergy*** *is defined to be the sum of the influence parameters* $s = h_{12} + h_{21}$.

We also make the following assumption to resolve cases in which the principal is indifferent between two contracts:

Assumption 1 *When computing the optimal contract, if the principal is indifferent between two arrangements A and B, and A is preferred by all agents over B, then A is chosen.*

The solution to the model and its properties are given by Proposition 1 below.

Proposition 1 *(Substitute production function, two agents.) (i) For all nonzero synergy, optimal efforts are equal:* $p_1^*(s) = p_2^*(s) \equiv p^*(s)$. *There exists a critical synergy level* $s^* > 0$ *such that*

$$p^*(s) = \begin{cases} \frac{2-\sqrt{4-3s}}{3s} & s \in (0, s^*) \\ 1 & s \geq s^*. \end{cases}$$

Optimal effort $p^*(s)$ *is strictly increasing on* $(0, s^*]$ *and explodes to 1 at* s^*. *When there is no synergy, any combination of efforts that sum to* $\frac{1}{2}$ *is optimal.*

(ii) Total wages given success, $w_1^* + w_2^*$, *and expected total wages* $\frac{p_1^*+p_2^*}{2}(w_1^* + w_2^*) = p^*(w_1^* + w_2^*)$ *are both increasing in* s *on* $(0, s^*]$.

(iii) Suppose synergy is subcritical. An increase in either influence parameter will lead to increases in optimal effort, total wages given success, and expected total wages.

(iv) Suppose synergy is subcritical. The more influential agent receives the higher wages upon success, i.e. $w_1^* > w_2^*$ *if and only if* $h_{12} > h_{21}$.

(v) Fix a subcritical synergy level. An increase in agent i's relative influence (i.e. increasing h_{ij} and lowering h_{ji} so that s is unchanged) increases both his relative and absolute wage. Specifically,

$$\frac{w_i^*}{w_j^*},\ \frac{w_i^*}{w_i^* + w_j^*},\ w_i^* \textit{ and } \ p^* w_i^* \quad \textit{all strictly increase.}$$

(vi) Suppose synergy is subcritical. An increase in h_{ij} increases w_i^* and $p^* w_i^*$. An increase in h_{ij} leads to an increase in w_j^* if and only if h_{ji} is sufficiently high. Specifically,

$$\frac{\partial}{\partial h_{ij}} w_j^* \begin{cases} > 0 & h_{ji} \in \left(\frac{1}{6p^*(s)}, s^* - h_{ij}\right) \\ = 0 & h_{ji} = \frac{1}{6p^*(s)} \\ < 0 & h_{ji} \in \left[0, \frac{1}{6p^*(s)}\right) \end{cases} . \tag{19}$$

Finally, $\frac{\partial}{\partial h_{ij}} p^* w_j^*$ is always positive.
(vii) The more influential agent receives the higher utility.

We now discuss the intuition behind and implications of each part of the Proposition. Part (i) shows that there are two effects that determine the agents' relative effort levels. Assume that agent i is more influential. On the one hand, i's greater influence tends to increase his optimal effort level, relative to j's, since his effort has greater cost reduction benefits. However, there is an effect in the opposite direction: the principal takes advantage of this cost reduction by increasing j's effort.

With our chosen functional forms, the two effects exactly cancel out. Mathematically, we can see in the principal's reduced-form objective function (18) that the cost saving due to synergy is $p_1 p_2 (h_{12} + h_{21})$. It thus depends on the product $p_1 p_2$ and is highest when $p_1 = p_2$. With different functional forms, these two effects may not exactly cancel out, and so the optimal effort levels will differ across the agents. The general takeaway from part (i) is that there are two opposing effects that determine the relative effort levels of the two agents, not that these two effects exactly cancel out. An implication of the second effect is that an agent's optimal effort level depends not only on his own parameters (his cost, productivity, and influence), but also positively on his colleagues' parameters (their influence). While an agent may have some control over his cost and productivity functions (e.g. by undergoing training), the model shows that the agent's effort (and, as we show in part (vi), his wage), are affected by parameters outside his control. Put differently, an agent's effort depends not only on his individual influence, but also the synergy which is common to both agents. To glean the intuition, the synergy can be thought of as an "echo" between the two agents – the influence of agent i on agent j raises i's optimal effort level, which reduces j's cost of effort, which raises j's optimal effort level, which, due to the influence of agent j on agent i, reduces i's cost of effort, which raises i's optimal effort level, and so on. In this process, it is the combination of influence parameters (here, their sum) that affects the optimal efforts of the two agents. Moreover, the expression for the cost saving $p_1 p_2 (h_{12} + h_{21})$ shows that, with our functional forms, the common synergy $s = h_{12} + h_{21}$ is a "sufficient statistic" for the equilibrium effort levels: an individual influence parameter does not matter other than through its effect on total synergy.

As the synergy s increases, the effort level of both agents increases: effort is more useful (due to its greater cost reduction benefit) and cheaper to implement (due to the cost reduction benefit). When the synergy crosses a threshold s^*, the optimal effort level jumps discontinuously to its maximum value of 1. When $s < s^*$, the echo between the two agents is dampening and the solution is interior. When $s > s^*$, the synergy is so strong that the echo is amplifying and the model "explodes".

Part (ii) states that wages increase with synergy. While intuitive, this result is far from automatic. With greater synergies, it is efficient to implement a higher effort level, which requires a higher wage holding all else equal. However, it seems that there is a counteracting effect – when synergies are higher, each agent's cost of effort is lower, and so a lower wage is required to implement a given effort level. Indeed, in a single-agent moral hazard model under risk neutrality and limited liability, the optimal contract involves paying the agent one-half of the firm's output, regardless of the agent's cost of effort, because these two effects exactly offset each other.

Here, wages are unambiguously increasing in the synergy parameter s. As discussed in the general model in Section 2, an agent's incentives to exert effort only depend on his wage and direct productivity, and are independent of his influence function. His influence is an externality that the agent does not internalize; the principal thus him a sharper contract to cause him to do so.

Part (ii) implies that total wages, as a fraction of output, will be higher in firms in which synergies are greater. Moreover, these higher wages come in the form of performance-sensitive pay. This is a potential explanation for why high equity incentives are sometimes given to rank-and-file employees, even if they have a small direct effect on output. High equity incentives are optimal if they have a significant impact on their colleagues' costs – for example, an efficient analyst in a private equity firm reduces the cost of a director going to a meeting by producing accurate briefing materials. Synergies are likely particularly high in small and young firms, where job descriptions are often blurred and interactions are frequent. This may explain why incentive-based compensation is particularly high in start-ups, even among low-level employees – for example, secretaries in Google were given equity and became wealthy ex post. Such "over-incentivization" is efficient if the secretaries significantly influence their colleagues. Hochberg and Lindsey (2010) document systematic evidence of broad-based option plans.[5]

Part (iii) follows naturally from parts (i) and (ii). Since an increase in one influence parameter, holding the other constant, raises the total synergy level s, it will raise the effort levels of both agents, total wages, and expected total wages. As discussed earlier, an increase in agent i's influence raises not only his optimal effort level, as his effort is now more useful to the principal, but also j's optimal effort level since j is now easier to incentivize.

While individual influence parameters do not affect effort or total wages, other than via their effect on the synergy, they do have an independent effect on the relative pay of each employee, as shown in part (iv). The more influential agent receives the higher wage. This result holds even though both agents exert the same effort, so the higher wage is not merely a "compensating differential" for disutility, and have the same direct productivity. Instead, the wage differential is driven purely by the desire for the influential agent to internalize his externality. Part (iv) leads to empirical predictions for within-firm differences in pay: more influential agents should receive higher wages, even if they perform the same tasks. For example, senior faculty are paid more than junior faculty even though they have the same formal job description; the former can reduce the latter's cost of effort through mentorship and guidance. While (iv) is a comparison, (v) is the related comparative static. If agent i's relative influence rises (h_{ij} increases and h_{ji} decreases so that s remains constant), i's wage goes up both in absolute terms and also relative to j's wage. The intuition is standard.

We now consider a rise in h_{ij} while holding h_{ji} constant. While part (iii) shows that an increase in i's influence parameter augments total wages, part (vi) studies the

[5]Note that our model can only explain equity compensation to rank-and-file employees that exert significant synergies on a sufficiently large number of people. If firms grant equity to non-synergistic employees, this is likely for alternative reasons already in the literature. Oyer (2004) justifies broad-based option plans from a retention perspective: options are worth more when employees' outside options are higher, persuading them to remain within the firm. Oyer and Schaefer (2005) find support for both this explanation and the idea that option compensation screens for employees with desirable characteristics. They do not test our synergy explanation, which has not been previously proposed to our knowledge. Bergman and Jenter (2007) present theory and evidence that option plans are used to take advantage of employees' irrational overvaluation of their firm's options.

effect on individual wages. It is clear that agent i's wage rises, since total wages rise (part (iii)) and i's share of total wages rises due to his greater relative influence (part (v)). However, there are two conflicting effects on j's absolute wage: total wages rise, but j's share of total wages falls. Part (vi) characterizes which force dominates when. If the principal held w_j^* constant, the rise in h_{ij} would increase j's effort because it reduces his marginal cost of effort. However, the principal need not hold w_j^* constant. She could choose to decrease j's wage and thus extract part of the "surplus" created by the rise in h_{ij} by paying j less; in return she accepts a smaller (but still positive) increase in j's effort. Put differently, since j's marginal cost of effort has fallen, it is cheaper to induce effort from him and she takes advantage of this by lowering his wage. Alternatively, she could increase j's wage and reinforce the increase in j's effort brought about by the rise in h_{ij}. Put differently, since it is cheaper to induce effort from j, she can take advantage of this by increasing j's effort even further (above and beyond the increase already occurring from the rise in h_{ij}) via a higher wage. The latter option is desirable if j's effort is particularly useful, i.e., if j's influence on i is particularly high.

The threshold level of h_{ji}, $\frac{1}{6p^*(s)}$, is decreasing in the common effort level and thus the common synergy – the higher the synergy, the greater the range of parameters h_{ji} under which j's wage increases. As explained earlier, the synergy creates an "echo" between the agents which amplifies the effect of changes in a parameter on the equilibrium. If the echo is strong enough, the increase in h_{ij} causes such a large increase in total wages that it outweighs the fall in j's share of the wage pool. While the change in j's absolute wage depends on h_{ji}, the expected wage $p^*w_j^*$ unambiguously rises (regardless of h_{ji}), due to the increase in the optimal effort level p^* from part (iii).

Finally, part (vii) compares the utility of the two agents. The more influential agent receives a higher wage, but also bears a higher cost since he is helped out less by his colleague. The Proposition shows that the first effect is stronger, and so the more influential agent receives the higher utility.

Given the importance of influence for pay, a natural question to ask is whether influence is inherent to a person, or to his position in the organization. Either may be true depending on the setting. The former will apply to a manager with strong leadership skills that encourage his colleagues to work hard, either through increasing their private benefit from working (employees enjoy working for an inspirational leader) or, equivalently, reducing their cost of doing so. Under this interpretation, influence can be considered a dimension of managerial talent. In talent assignment models with moral hazard (e.g. Edmans, Gabaix, and Landier (2009)), talent affects productivity and is thus internalized by the agent without the need to modify the contract; here, influence affects colleagues' cost functions and so affects the optimal wage. The latter will apply to an employee who occupies a central position in an organization that allows him to exert influence. He may be in this position through historical accident or entrenchment, but even though his high wages are a result of luck (occupying a central position) rather than rents to talent, they are efficient.

4 The Three-Agent Model

In the two-agent model, there was a single synergy component. Thus, both agents shared the same synergy, and as a result exert the same effort level. We now extend the model to three agents, which allows for synergies to vary across agents: in particular, one agent may enjoy synergies with both of his colleagues, but his colleagues may not enjoy synergies with each other. The production function (1) now becomes:

$$\Pr(r = 1) = \frac{p_1 + p_2 + p_3}{3}. \tag{20}$$

and we continue to assume a quadratic individual cost function, which is now given by:

$$h_i(p_i) = \frac{1}{6} p_i^2.$$

Differentiating agent i's utility function (5) gives his first-order condition as:

$$w_i(p_i) = p_i \left(1 - \sum_{j \neq i} h_{ji} p_j \right), \tag{21}$$

and plugging this into the principal's objective function (3) gives her reduced-form maximization problem as:

$$p_1^*, p_2^*, p_3^* \in \arg \max_{p_1, p_2, p_3 \in [0,1]} \frac{(p_1 + p_2 + p_3)}{3} \left(1 - (p_1 + p_2 + p_3) + A p_1 p_2 + B p_1 p_3 + C p_2 p_3\right), \tag{22}$$

where

$$A = h_{12} + h_{21} \quad B = h_{13} + h_{31} \quad C = h_{23} + h_{32}.$$

We define the following terms:

Definition 2 *The **synergy profile*** $\mathbf{s}$ *is defined to be the vector* (A, B, C)*. The quantities* A*,* B *and* C *are the **synergy components** of the synergy profile. The **size** of* $\mathbf{s}$ *is defined to be* $s = ||(A, B, C)||$.

Quantity A is the analog of the synergy scalar s in the two-agent model: it measures the sum of the influence that agents 1 and 2 exert on each other, and B and C are defined analogously for agents 1 and 3 and agents 2 and 3, respectively. In a three-agent model, there are three relevant synergy components between each of the three pairs of agents, which together form the synergy profile $\boldsymbol{s}$.

The solution to the model is given by Proposition 2 below for the case of an interior solution, and Proposition 3 for the case of a boundary solution.

Proposition 2 *(Substitute production function, three agents, interior solution.) (i) Suppose the synergy profile* $\mathbf{s}$ *is strictly nonzero and the optimal effort profile* $\mathbf{p}^*(\mathbf{s}) = (p_1^*(\mathbf{s}), p_2^*(\mathbf{s}), p_3^*(\mathbf{s}))$ *is interior. Then we have:*

$$Ap_2^*(\mathbf{s}) + Bp_3^*(\mathbf{s}) = Ap_1^*(\mathbf{s}) + Cp_3^*(\mathbf{s}) = Bp_1^*(\mathbf{s}) + Cp_2^*(\mathbf{s}) \tag{23}$$

which implies

$$\frac{p_1^*(\mathbf{s})}{p_2^*(\mathbf{s})} = \frac{C}{B}\frac{A+B-C}{A+C-B} \quad ; \quad \frac{p_2^*(\mathbf{s})}{p_3^*(\mathbf{s})} = \frac{B}{A}\frac{A+C-B}{B+C-A} \quad ; \quad \frac{p_3^*(\mathbf{s})}{p_1^*(\mathbf{s})} = \frac{A}{C}\frac{B+C-A}{A+B-C}. \tag{24}$$

In particular, interior optimal effort profiles occur only when each synergy component is strictly smaller than the sum of the other two. Moreover, the optimal effort ratios are homogenous of degree 0 in A, B *and* C*. Therefore the direction of the synergy profile is sufficient to determine the direction of the optimal effort profile provided it is interior.*

(ii) Fix a direction of the synergy profile such that each component is strictly smaller than the sum of the other two. There exists a critical synergy size threshold s^* *such that, if* s *is subcritical then the optimal effort profile is interior, and the size of the optimal effort profile is a strictly increasing function of synergy size.*[6] *At the critical synergy size* s^**, the optimal effort profile explodes so that at least one agent is now applying effort 1.*

(iii) Total wages given success and expected total wages are strictly increasing in s *up to the critical synergy size* s^**.*

(iv) Fix a synergy profile such that the optimal effort profile is interior. An increase in agent i*'s relative influence (i.e., increasing at least one element of* $\{h_{ij}\}_{j\neq i}$ *and*

[6] Recall that part (i) implies that, in this interval, the direction of the optimal effort profile is fixed.

decreasing some elements of $\{h_{ji}\}_{j\neq i}$ *so that* $\mathbf{s}$ *is unchanged) increases both his relative and absolute wage. Specifically,*

$$\frac{w_i^*}{\sum_j w_j^*},\ w_i^* \text{ and } p^* w_i^* \text{ all strictly increase,}$$

and

$$\frac{w_i^*}{w_j^*} \text{ weakly increases for all } j \text{ and strictly increases at least one } j.$$

Proposition 3 *(Substitute production function, three agents, boundary solution.) Suppose there is a single synergy component that is greater than the sum of the other two. Then the efforts exerted by the two agents who have the largest synergy with each other are equal and the other agent does not exert effort. The size of the other two synergy components has no effect on the optimal effort profile and the model is isomorphic to the 2-agent model.*

Combining the results of Propositions 2 and 3 gives the full solution to the model as Theorem 2, the key result of this section:

Theorem 2 *The optimal effort profile is summarized in Figure 1.*

Corollary 1 *Suppose the influence between any pair of agents is symmetric. That is for each* $i \neq j$, $h_{ij} = h_{ji}$. *Then when the optimal effort profile is in the interior, the ratios of optimal wages coincide with the ratios of optimal efforts.*

We now discuss the intuition behind and implications of each of the above results. Part (i) of Proposition 2 states that the ratio of the optimal effort levels only depends on the relative size of the different synergy components A, B and C, and not their absolute magnitude. Thus, a proportional increase in each synergy component will augment each effort level to the same degree, leaving the ratios unchanged.

Part (ii) states that, if the synergy profile s is sufficiently small, and the synergy components are balanced so that no single component exceeds the sum of the other two, the optimal effort profile is interior. Analogous to part (i) of Proposition 1, when synergy size increases, effort by each agent becomes both more useful and easier to incentivize, and so it is optimal for the principal to implement a higher effort profile. When synergies become sufficiently strong, the principal implements the maximum effort level of 1 for at least one agent.

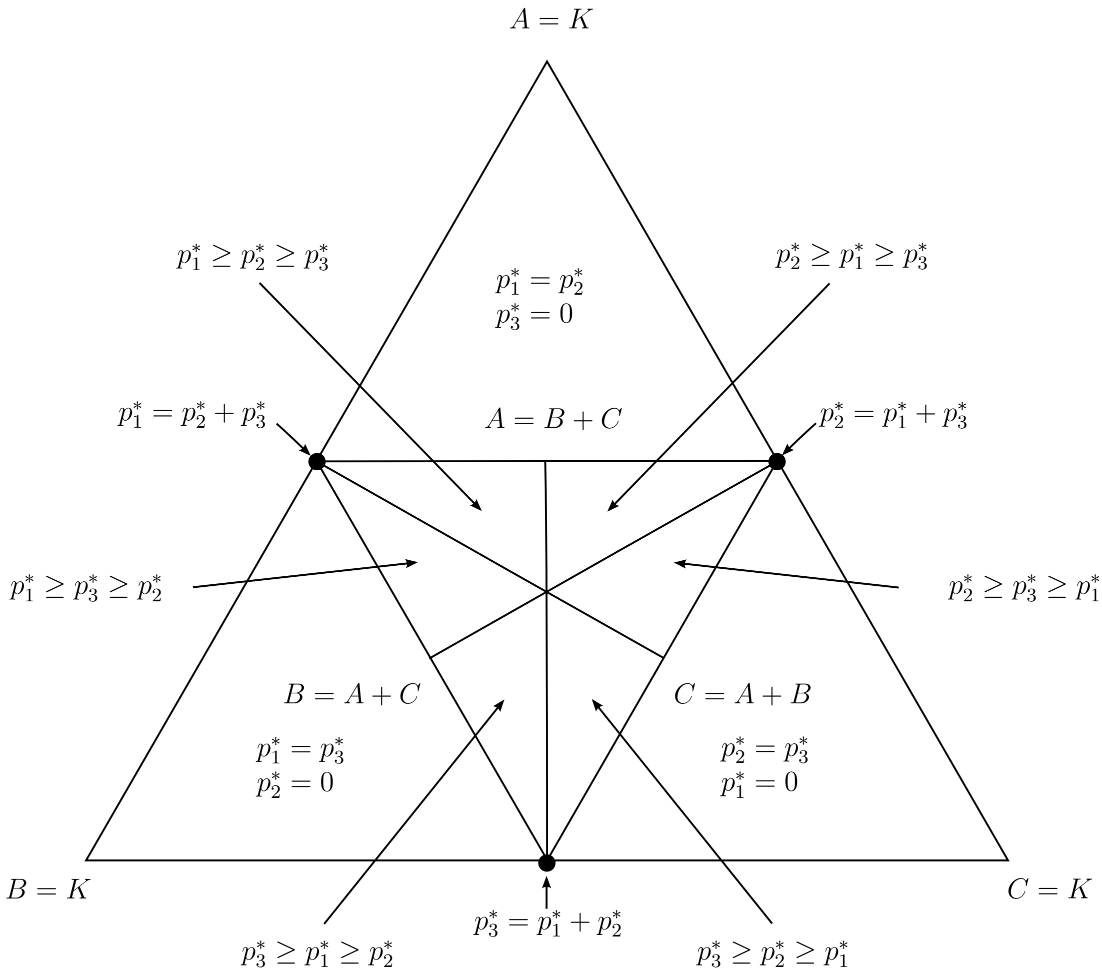

Figure 1: The $A + B + C = K$ simplex where $K > 0$ is some constant.

The simplex in Figure 1 fixes the sum of the synergy components $A + B + C$ at a constant K and studies the effect of changing their relative level. The middle triangle (bounded by the three dots) in Figure 1 illustrates the case of an interior effort profile summarized by Proposition 2. For an interior effort profile, all three synergy components matter for the relative effort levels. For example, if and only if $B > C$ (i.e. the left-hand side of the triangle), we have $p_1 > p_2$: since agent 1 generates more synergies with agent 3 than does agent 2, 1 exerts more effort than 2.

Note that it is the *total* synergy between agents 1 and 3 (relative to the total synergy between 2 and 3) that determines the relative values of p_1 and p_2, not 1's unidirectional influence on 3, h_{13} (relative to 2's unidirectional influence on 3, h_{23}). It may seem that p_1 should only depend on h_{13} (and not h_{31}) as only the former affects the usefulness of 1's effort. However, when h_{31} rises, 1's cost function is lower and so it is cheaper to implement a higher p_1. The intuition is similar to the two-agent model: it is the total synergy that matters, not one's individual influence parameter. Hence, we have

$p_1 > p_2$ when $B > C$, rather than when $h_{13} > h_{23}$.

On the one hand, this result extends the principle in the two-agent case, that the optimal effort level depends on the common synergy, not the individual influence parameters. On the other hand, the result contrasts the two-agent case, since all agents no longer exert the same effort level. In the two-agent case, there is only one synergy component and so one common effort level. Here, the existence of three synergy components allows for asymmetry in effort levels between the three agents. However, while there are individual effort levels, they still only depend on the common synergy components, not the individual influence parameters.

A natural application of the model is where one synergy component (say C) is close to zero. For example, agent 1 is a CEO who shares synergies with two division managers, agents 2 and 3, but they share few synergies with each other. Figure 1 shows that agent 1 exerts the highest effort. Essentially, agents 2 and 3 can be aggregated, and their combined effort level is close to the effort exerted by agent 1.

Proposition 3 considers a boundary effort profile. It states that, if one synergy component exceeds the sum of the other two, then the model collapses to the two-agent model of Proposition 1. Intuitively, if the synergy between two agents is sufficiently strong, then only these two matter for the principal – she ignores the third agent and induces zero effort from him. This "corner" result (captured by the three triangles that surround the middle triangle in Figure 1) is striking because the third agent has the same direct effect on the production function (20) as the other two, yet is completely ignored. Moreover, it means that even if there is no change to the synergies between agent 3 and his colleagues, an increase in the synergies between agents 1 and 2 can lead to him being excluded (either fired from the team if he was previously a member, or not hired in the first place). Thus, 3's participation depends not only on his own synergy parameters, but also on parameters that have no direct relevance to him.

That the third agent is completely excluded seems surprising, since we have a convex cost function and the marginal cost of effort for agent 3 is zero. It may thus seem cheaper to increase agent 3's effort from 0 to ε than to increase the effort of agents 1 and 2 from $p - \varepsilon$ to p. The key to the intuition is that it is not the marginal cost of effort that is relevant (which is indeed lower for 3 than 1 and 2) but the marginal increase in wage that the principal needs to offer to induce effort. This intuition can be seen by differentiating the principal's objective (22) with respect to p_3, which yields the first-order condition:

$$\frac{1}{3}\left(1 - (p_1 + p_2 + p_3) + Ap_1p_2 + Bp_1p_3 + Cp_2p_3\right) + \frac{(p_1 + p_2 + p_3)}{3}\left(-1 + Bp_1 + Cp_2\right).$$

The first term is the marginal increase in output from increasing p_3, multiplied by the share that the principal retains after paying out wages. The exact same term also appears in the first-order conditions with respect to p_1 and p_2.

The $\frac{(p_1+p_2+p_3)}{3} - 1$ term is the additional wage the principal must pay agent 3 to implement this higher effort level in the absence of synergies. Again, the exact same term also appears in the first-order conditions with respect to p_1 and p_2: it costs exactly the same to induce additional effort from agent 3 as it does to induce additional effort from 1 and 2. This result seems surprising given the convex cost of effort, but can be understood by considering agent i's maximization problem. He solves:

$$p_i \in \arg\max_p \frac{(p_1+p_2+p_3)}{3} w_i - \frac{1}{6} p_i^2 \left(1 - \sum_{j \neq i} h_{ji} p_j\right)$$

which yields

$$w_i(p_i) = p_i \left(1 - \sum_{j \neq i} h_{ji} p_j\right)$$

i.e. equation (21). The wage that the principal needs to pay agent i is linear, rather than convex, in the effort she wishes to induce. It is true that she has to pay agents 1 and 2 more than 3 in *absolute* terms, due to the convex cost of effort, but the *marginal* increase in the wage for inducing more effort is the same for all agents. Even though the marginal cost of additional effort is high for agents 1 and 2, due to the convex cost function, the marginal benefit for them is also high since they are already receiving a wage (and thus a share of output); thus only a small increase in the wage is required. For agent 3, the marginal cost of additional effort is low, but the marginal benefit is also low since he currently has no share of output. Thus, in the absence of synergy, it is just as costly to increase effort by agent 3 from 0 to ε as it is to increase effort by agents 1 and 2 from $p - \varepsilon$ to p.

The final term is the synergy term, which reduces the increase in wage required to induce greater effort. This term does differ across agents. For the three agents, these terms are:

$$X_1 = \frac{(p_1+p_2+p_3)}{3}(Ap_2 + Bp_3) = \frac{(p_1+p_2+p_3)}{3} Ap^* \text{ at } (p^*, p^*, 0)$$
$$X_2 = \frac{(p_1+p_2+p_3)}{3}(Ap_1 + Cp_3) = \frac{(p_1+p_2+p_3)}{3} Ap^* \text{ at } (p^*, p^*, 0)$$
$$X_2 = \frac{(p_1+p_2+p_3)}{3}(Bp_1 + Cp_2) = \frac{(p_1+p_2+p_3)}{3}(B+C)\,p^* \text{ at } (p^*, p^*, 0)$$

For $p_3 = 0$ to be optimal, it is not sufficient merely for $A > B$ and $A > C$ (i.e. the synergy component between agents 1 and 2 to be higher than the other two synergy components): since p_1 and p_2 are high, the synergy benefit from inducing more effort from agent 3, X_3 can exceed X_1 and X_2 even if $B < A$ and $C < A$. Only if $A > B+C$, i.e. the synergy component between agents 1 and 2 is not only greater than the two other synergy components, but also greater than their sum, is X_3 lower than X_1 and X_2 and so it is more efficient to induce effort from the other two agents. Due to the strong synergy, raising the effort of agents 1 and 2 "echoes" many times and is thus more effective than raising p_3. Another way to view the intuition is that increased synergy between 1 and 2 raises the value of the firm, and thus the cost to the principal of giving 3 equity to induce effort from him.

The above result has interesting implications for the optimal composition of a team. If two agents exhibit sufficiently high synergies with each other, there is no benefit in adding a third agent to the team, even if he has just as high direct productivity as the existing two agents and has strictly positive synergies with them. Moreover, the agents can be interpreted as divisions of a firm, in which case Proposition 3 has implications for firm boundaries. If two divisions exhibit sufficiently strong synergies with each other, it may be optimal to divest a third division even if it exhibits strictly positive synergies with the first two. Conversely, it may be optimal for a two-division firm not to acquire a third division even if it would generate strictly positive synergies, if those synergies are low relative to those enjoyed by the two existing divisions. Conventional wisdom is that any division that enjoys positive synergies should be included within a firm. Here, it is relative, not absolute, synergies that determine firm boundaries. The empirical implication is that a decision to divest (or not acquire) a division might not be driven by the low synergies generated (or potentially generated) by this division, but rather by the strong synergies between other divisions.[7]

While in Proposition 2, all three synergy components matter for the optimal effort profile, in Proposition 3 only the largest synergy component matters and the other two are irrelevant. Within the middle triangle, the relative size of B and C affects the relative size of p_1 and p_2, as discussed earlier. In the top triangle (where $A > B + C$), we have $p_1 = p_2$ regardless of the relative size of B and C. The synergy between agents

[7]This implication assumes that it is not possible to compensate the division based only on its own performance, but only according to the performance of the overall conglomerate. While divisional profit measures are typically available, there is only a stock price for the overall conglomerate. The stock price incorporates many additional pieces of information than profits, such as growth prospects; consequently, managerial pay is typically much more sensitive to the stock price than to profits (see, e.g., Murphy (1999)).

1 and 2 is so important that their individual synergies with 3 become irrelevant.

We now turn from the optimal effort profile to the optimal wage profile. Part (iii) of Proposition 2 is analogous to part (ii) of Proposition 1: total wages depend on the total synergy across all agents. While total synergy determines total wages, the influence parameters determine relative wages: part (iv) of Proposition 2 is analogous to part (iv) of Proposition 1. An increase in one agent's influence augments his wage in both absolute and relative terms. Moreover, if the influence parameters are symmetric across all pairs of agents, the entire wage profile can be fully solved: Corollary 1 states that the ratios of optimal wages coincides with the ratios of optimal effort.

The model can thus explain why CEOs earn significantly more than other senior managers. Bebchuk, Cremers, and Peyer (2011) argue that this is due to inefficient rent extraction by the CEO, but our theory suggests that it may be optimal since the centrality of the CEO leads to him exhibiting greatest synergies. High wages for the CEO lead his colleagues to anticipate that he will work harder, in turn inducing them to exert greater effort themselves.[8] More generally, the model shows that a CEO's wage depends on the scope of the firm under his control, i.e. the number of agents (or divisions) with which he exhibits synergies and the strength of these synergies. The increase in communication technology over the past few decades has plausibly increased the CEO's influence, consistent with the rise in CEO pay over the same period. Talent assignment models argue that CEO pay depends on firm size (e.g. Gabaix and Landier (2008), Terviö (2008)), which is typically measured by an accounting variable such as total assets or profits. Our theory suggests that the relevant measure of firm size is the scope and depth of the CEO's synergies. Thus, the CEO of a large firm in which the divisions operate independently (e.g. a holding company) may be paid less than the manager of a small firm where there are strong synergies (e.g. a start-up).

5 Extensions and Discussion

This section contains additional analyses to the core model of Sections 3 and 4.

[8]Kale, Reis, and Venkateswaran (2009) study another reason for why high pay for the CEO may be efficient – to provide tournament incentives for other senior managers. They find that the pay differential between the CEO and other senior managers is positively related to firm performance. Aggarwal, Fu, and Pan (2011) use the pay differential as a proxy for non-CEO executives to monitor the CEO, as in the theory of Acharya, Myers, and Rajan (2011).

5.1 Negative Influence Parameters

This subsection extends the model to the case where the influence parameters h_{ij} can be negative. We start with the two-agent model and then move to the three-agent model.

5.1.1 The Two-Agent Model

Recall the principal's reduced-form maximization problem is given by:

$$p_1^*, p_2^* \in \arg\max_{p_1,p_2} \frac{p_1+p_2}{2}\left(1-(p_1+p_2)+p_1p_2(h_{12}+h_{21})\right).$$

There are thus two cases to consider.

Case 1. $h_{12} > 0 > h_{21}$, and $h_{12}+h_{21} > 0$.

By inspecting the maximization problem, we can see that the solution only depends on the total synergy s and not the individual influence parameters h_{ij}. Since we have $s > 0$, we are in the case of the core model and so Proposition 1 holds.

Case 2. $h_{12}+h_{21} < 0$.

Since we now have $s < 0$, by inspecting the maximization problem we can see that the solution requires $p_1^* p_2^* = 0$ and so one agent exerts zero effort. Since both agents have the same direct productivity, it does not matter which agent this is. Without loss of generality, assume that $p_2^* = 0$. Then the principal solves:

$$p_1^* \in \arg\max_{p_1} \frac{p_1}{2}\left(1-p_1\right).$$

This is a single-agent model. The solution is standard, and is given by Proposition 4 below:

Proposition 4 *(Substitute production function, two agents, negative synergy.) Suppose that the total synergy s is negative. Then only one agent exerts strictly positive effort; without loss of generality, assume this is agent 1. The analog of Proposition 1 is as follows:*

(i) The optimal effort levels are given by $p_1^*(s) = \frac{1}{2}$, $p_2^*(s) = 0$.

(ii) The wage levels are given by $w_1^* = \frac{1}{2}$ and $w_2^* = 0$, and are independent of s as long as $s < 0$.

(iii) An increase in either influence parameter has no effect on effort and wages as long as $s < 0$.

(iv) Since the principal is indifferent over which agent has the zero effort and wage level, it is possible to have $w_1 > w_2$ *for* $h_{12} < h_{21}$.

(v) For a fixed $s < 0$*, changes in agent i's relative influence have no effect.*

(vi) As long as $s < 0$*, changes in agent i's absolute influence have no effect.*

(vii) The agent who is exerting effort has the higher utility. Since the principal is indifferent over which agent has the zero effort and wage level, it is possible that this is the less influential agent.

We can summarize the above results as follows. Case 1 shows that, as long as the total synergy is positive, the core model's results continue to hold in the case where one influence parameter is negative. It may seem surprising that the principal chooses to hire (i.e., induce strictly positive effort from) an agent that exert negative influence. However, again, it is the total synergy that matters for whether both agents exert effort, so it does not matter if one influence parameter is negative as long as the total synergy is positive. Case 2 shows that, if total synergy is negative, the principal only wishes to hire one agent, and the individual influence parameters are irrelevant for the choice of agent.

5.1.2 The Three-Agent Model

In the two-agent model, the solution depended on whether the total synergy (rather than the individual influence parameters) was positive or negative. In the three-agent model, the solution depends on whether the synergy components are positive or negative. Without loss of generality, we will assume that A is the largest synergy component, followed by B and then C. There are four cases to consider:

Case 1. $A > B > C > 0$.

If each synergy component is positive, we are in the case of the core model and Propositions 2 and 3 continue to hold.

Case 2. $A > B > 0 > C$.

Here, one of the synergy components is negative. This ensures that there is a single synergy component that is greater than the sum of the other two: $A > B + C$. We thus obtain the corner solution of Proposition 3. Only the two agents who have the largest synergy with each other exert effort, and the problem reduces to the 2 agent model.

Case 3. $A > 0 > B > C$

This case is similar to Case 2 in that we have $A > B + C$. We thus obtain the corner solution of Proposition 3.

Case 4. $0 > A > B > C$.
In this case, only one agent exerts effort. Since all three agents have the same direct productivity, it does not matter which agent this is. Without loss of generality, assume that $p_2^* = p_3^* = 0$. We are in a single agent model where $p_1^* = \frac{1}{2}$ and the analogy of Proposition 4 applies.

5.2 Discussion: Cost Synergies

A key feature of our model is that an agent's effort reduces the marginal cost of effort of his colleague. As mentioned before, this can be interpreted as an agent's effort increasing the marginal private benefit that the colleague derives from his own effort. This feature generates the synergies in our model. To what extent is this different from instead modeling complementarities in the production function, i.e., that an agent's effort increases the marginal productivity of his colleagues' effort?

In a single-agent model with separable utility, changing the agent's marginal productivity by multiplying the production function by a constant factor is indeed isomorphic to changing his marginal cost by dividing the cost function by the same factor. However, in a multi-agent world, cost synergies are fundamentally different to production complementarities because the former are a true externality, but the latter are not. Since contracts are contingent upon output but cannot be made contingent on effort costs, agents naturally internalize the effects of their efforts on production but not on costs. To illustrate, consider a two-agent model with production complementarities but no cost synergies. The production function is given by

$$\Pr(r = 1) = a\frac{p_1 + p_2}{2} + b\sqrt{p_1 p_2},$$

where b parameterizes the complementarity. The principal maximizes:

$$\left(a\frac{p_1 + p_2}{2} + b\sqrt{p_1 p_2}\right)(1 - w_1 - w_2), \tag{25}$$

and agent i's objective function is:

$$\left(a\frac{p_i + p_j^*}{2} + b\sqrt{p_i p_j^*}\right) w_i - \frac{1}{4}p_i^2. \tag{26}$$

The solution is given in Proposition 5 below.

Proposition 5 *(Complementaries in production function, two agents.)*

(i) The optimal effort profile is $p_1^* = p_2^* = \frac{a+b}{4}$.

(ii) To implement an arbitrary effort profile (p_1, p_2), *the principal offers wages* $w_1 = \frac{p_1}{a+b\sqrt{\frac{p_2}{p_1}}}$ *and* $w_2 = \frac{p_2}{a+b\sqrt{\frac{p_1}{p_2}}}$.

(iii) At the optimal effort profile (p_1^*, p_2^*), *the principal offers wages* $w_1^* = w_2^* = \frac{1}{4}$.

The economics are as follows. Part (i) shows that the optimal effort profile is increasing in the production complementarity b, just as in the core model where it was increasing in the cost synergy s. Part (ii) demonstrates a counteracting effect absent from the core model. For a given effort level p_i, the wage required to implement this effort level is decreasing in b. Since the production complementarity term $b\sqrt{p_1 p_2}$ is shared across agents, an increase in the production complementarity P_{ij} (through raising b) also augments agent i's marginal productivity P_i, as can be seen from i's objective (26).[9] Since agent i takes into account his increased productivity, he does not need as high a wage to induce a given effort level. Indeed, part (iii) shows that, under the current functional forms, these two effects exactly cancel out: while a rise in b increases the optimal effort level, it also reduces the wage required to induce a given effort level, so the overall wage is unchanged. Put differently, since i fully internalizes his increased productivity, he will exert the new, higher, optimal effort level even under the original contract.

With other functional forms, it may be that the two effects do not exactly cancel out. The point that is robust to functional forms is that the second effect always exists – the wage always depends directly on the production complementarity b, because it affects the agent's marginal benefit of effort and is thus internalized – not necessarily that it exactly cancels out the first effect (that higher b increases the effort level). Indeed, in the general model of Section 2, a change in the production function $P(\cdot)$ to $\widetilde{P}(\cdot)$ (to increase the production complementarity P_{12}) will also change i's first-order condition from

$$w_i P_i(p_i, p_j) - g_i'(p_i)\, \Pi_{j \neq i} h_{ji}(p_j)$$

to

$$w_i \widetilde{P}_i(p_i, p_j) - g_i'(p_i)\, \Pi_{j \neq i} h_{ji}(p_j),$$

[9]With cost synergies, we were able to undertake a rescaling that changed only the cross-partial and kept agent j's optimal effort level constant if p_i were held constant at p_i^*. This was possible because cost functions are specific to an agent and so can be changed individually: i's influence could be changed while keeping j's first-order conditions constant under the old p_i^*. In contrast, the production function is common to both agents. Thus, it is not possible to change the production function (even with a rescaling) without affecting j's first-order conditions. Thus, j's optimal effort level would change even if p_i were held constant at p_i^*.

and is thus internalized.

In contrast, in the model with cost synergies only, the principal maximizes:

$$\frac{p_i + p_j}{2} \left(1 - p_i \left(1 - h_{ji} p_j\right) - p_j \left(1 - h_{ij} p_i\right)\right) \tag{27}$$

and agent i's objective function is:

$$\frac{p_i + p_j^*}{2} w_i - \frac{1}{4} p_i^2 \left(1 - h_{ji} p_j^*\right). \tag{28}$$

To implement an arbitrary effort profile (p_i, p_j), the principal offers agent i a wage of

$$w_i = p_i (1 - h_{ji} p_j). \tag{29}$$

Differentiating the principal's objective (27) with respect to p_i shows that, when i's influence h_{ij} increases, the principal wishes to implement a higher effort level p_i. However, inspecting the agent's objective (28) shows that agent i does not consider his influence on agent j, h_{ij}, when choosing his effort level: this term does not appear in his objective function (unlike the production complementarity b in (26)). An increase in cost influence has no effect on agent i's direct productivity: the only effect is on agent j's cost of effort which is noncontractible, and so i does not consider it when making his own effort choice. Unlike the production complementarity term which is shared across agents, the cost function $\frac{1}{4} p_j^2 \left(1 - h_{ij} p_i\right)$ is specific to agent j, so a change in the cost synergy C_{ij} (through raising h_{ij}) does not affect agent i's marginal cost C_i. Therefore, the wage required to implement a given effort profile does not depend on the cost influence h_{ij} (see equation (29)).

For any functional form, the second effect in the production complementarities model – that an increase in production complementarity reduces the wage required to implement a given effort level – does not exist in a model of cost synergy. In the general model of Section 2, agent i's first-order condition is

$$w_i P_i(p) - g_i' \left(p_i\right) \Pi_{j \neq i} h_{ji} \left(p_j\right),$$

and unaffected by a change in his influence function $h_{ij}(\cdot)$ to $\widetilde{h}_{ij}(\cdot)$. Thus, the increase in i's influence is a true externality, and the principal must increase i's wage to cause him to internalize it.

Even though cost and production synergies are fundamentally different from a modeling standpoint, in that the latter but not the former are internalized by an agent, they are similar in the economic idea that they represent. With production synergies, effort by one agent increases the marginal productivity of his colleague, for a given unit cost. With cost synergies, effort by one agent reduces the marginal cost of his colleague, for a given unit productivity. Thus, our model continues to capture the same economic idea that synergies improve a colleague's productivity-to-cost ratio. The difference from a modeling standpoint is that improving a colleague's productivity-to-cost ratio through changing the production function augments the agent's own productivity, since the production function is shared across agents, but improving a colleague's productivity-to-cost ratio through changing his cost function has no effect on the agent's own productivity, since the cost function is specific to an agent. (Similarly, under the interpretation that increased influence augments a colleague's private benefit of effort, private benefit is also specific to an agent).

The analysis of Section 2 showed that the effect of increasing agent 1's influence on his optimal effort level is independent of production complementarities. In Appendix B, we analyze the model of Sections 3 and 4 under the case of production complementarities as we well as cost synergies. We show that the addition of production complementarities does not change the implications generated by cost synergies.

6 Conclusion

This paper has studied the effect of synergies on optimal effort levels and wages in a team-based setting. We model synergies as effort by one agent reducing the cost, or increasing the private benefit, of effort by a colleague. This is a fundamentally different notion of synergy to complementarities in the production function and leads to a number of new results. With synergies, the principal considers two additional forces when determining an agent's optimal effort level – the indirect effect of greater effort on his colleague's cost function, and the impact on his colleague's marginal productivity. While the sign of the second effect depends on the nature of the production function, we show that an increase in an agent's influence – captured by a more negative cross-partial derivative of his colleague's cost function – increases his optimal effort level, regardless of the production function. In addition, if there are no production complementarities, and the agent in question is not influenced by any of his colleagues, his wage unambiguously increases. Since the synergy is a true externality, the agent does not take it into account when choosing his effort level, and so the principal must "over-incentivize" him to cause him to internalize this effect.

With the standard setup of a linear production function and quadratic cost function, additional results arise. In a two-agent model, an agent's optimal effort level depends not only on his productivity, cost, and influence, but also his colleague's influence – it depends not only on individual influence parameters, but also the common synergy. Wages differ across agents, even though both agents exert the same effort level and have the same direct impact on output, with the more influential agent receiving higher pay. Total wages increase with the total level of synergy, suggesting that it may be optimal to grant rank-and-file employees strong equity incentives, even if their direct effect on output is low. An increase in one agent's influence parameter augments his own effort and pay, but raises his colleague's pay if and only if his colleague is sufficiently influential.

With three agents, optimal effort levels differ and depend on the total synergies an agent enjoys with his colleagues rather than his unidirectional influence. If synergies between two agents are sufficiently strong, it is optimal for the principal to focus entirely on these agents and ignore the third. This result has implications for the optimal composition of a team and optimal firm boundaries – if synergies between two agents (divisions) become sufficiently strong, it is efficient to discard the third agent (division) even if his (its) own parameters do not change. Agents that exert synergies over a greater number of colleagues receive higher pay, consistent with the wage premia CEOs enjoy over divisional managers.

A Proofs

We first start with a maximization problem which we will make repeated use of in these proofs. Consider the following maximization problem where $a, b \geq 0$:

$$\max_{x \in [0,1]} x(1 - bx + ax^2).$$

Let $x^*(a, b)$ denote the set of argument solutions.

Lemma 1 *(i) If $b \leq \frac{1}{2}$, then $x^*(a, b) = 1$.*

(ii) If $b > \frac{1}{2}$, then there exists a threshold $a^*(b) > 0$ such that

$$x^*(a,b) = \begin{cases} \frac{b-\sqrt{b^2-3a}}{3a} & a < a^*(b) \\ \{\frac{b-\sqrt{b^2-3a}}{3a}, 1\} & a = a^*(b) \\ 1 & a > a^*(b) \end{cases}.$$

Proof. We first define some notation. Let $U(x, a, b) = x(1 - bx + ax^2)$ and $x^{loc}(a, b) = \frac{b-\sqrt{b^2-3a}}{3a}$.

First let $b \leq \frac{1}{2}$. If $a = 0$, it is clear that $x^*(0, b) = 1$. If $a > 0$, then

$$\frac{\partial}{\partial x} U(x, a, b)|_{x=1} = 1 - 2bx + 3ax^2|_{x=1} = 1 - 2b + 3a > 0 \tag{30}$$

To show $x^*(a, b) = 1$, it suffices to show there is no local maximum of $U(x, a, b)$ on $(0, 1)$. By the quadratic formula, a local maximum exists (anywhere) if and only if $b^2 - 3a = 3a(b \cdot \frac{b}{3a} - 1) > 0$. Since $b \leq \frac{1}{2}$, this implies $\frac{b}{3a} > 2$. In addition, $\frac{b}{3a}$ is the inflection point of $U(x, a, b)$. Since $U(x, a, b)$ is a positive cubic, the inflection point lies above the local maximum. Thus, since $\frac{\partial}{\partial x} U(x, a, b) > 0$ for $x = 1$ (from (30)), and the inflection point is not reached until $x = \frac{b}{3a} > 2$, the local maximum must be between $x = 1$ and $x = \frac{b}{3a}$. Thus, we must also have $\frac{\partial}{\partial x} U(x, a, b) > 0$ for all $x < 1$. Thus, there is no local maximum of $U(x, a, b)$ on $(0, 1)$.

Now consider $b > \frac{1}{2}$. We have the following facts:

Fact 1: $x^{loc}(a, b)$ is strictly increasing in 1 on $[0, \frac{b^2}{3}]$. This follows from the fact that $b - \sqrt{b^2 - 3a}$ is convex while $3a$ is linear and both are equal to zero when $a = 0$.

Fact 2: By the envelope theorem,

$$\frac{\partial}{\partial a} U\left(x^{loc}(a, b), a, b\right) = \left[x^{loc}(a, b)\right]^3 < 1 \;\text{ when }\; x^{loc}(a, b) < 1$$

Fact 3: On the other hand,

$$\frac{\partial}{\partial a} U(1, a, b) = 1$$

Fact 4: For all sufficiently low 1, $x^*(a, b) = x^{loc}(a, b)$. To see this, notice since $\lim_{a \downarrow 0} x^{loc}(a, b) = \frac{1}{2b} < 1$, so for all sufficiently low 1, the local maximum is in the interval $(0, 1)$. Of course when $a = 0$, the local maximum is the global maximum. By continuity, the fact is true.

Clearly, whenever $x^{loc}(a, b) > 1$ or does not exist, then $x^*(a, b) = 1$. Therefore, suppose $x^{loc}(a, b) \leq 1$ and exists. Fact 1 implies that the set of 1 that satisfy these two conditions is of the form $[0, \tilde{a}]$ where $\tilde{a} \leq \frac{b^2}{3}$. We wish to show that $U\left(x^{loc}(a, b), a, b\right)$ and $U(1, a, b)$ satisfy the single crossing property on the interval $[0, \tilde{a}]$. $\tilde{a}$ is the upper bound on the interval of 1's such that $x^{loc}(a, b) \leq 1$ and exists. Thus, there are two cases to consider. First, we could have $x^{loc}(\tilde{a}, b) = 1$, in which case the functions $U\left(x^{loc}(a, b), a, b\right)$ and $U(1, a, b)$ cross at $a = \tilde{a}$. Second, we could have $x^{loc}(\tilde{a}, b) < 1$. Note that at $a = \tilde{a}$, the function $U(x, a, b)$ must have a single critical point. If it had two critical points, we could increase 1. An increase in 1 "flattens" out the cubic by bringing the value of the local minimum and local maximum closer, but since there are two critical points to begin with, this can be done without violating the requirement that at least one critical point, $x^{loc}(a, b)$, exists. An increase in 1 also raises $x^{loc}(\tilde{a}, b)$ (from Fact 1), but since $x^{loc}(\tilde{a}, b) < 1$, this can be done without violating the constraint that $x^{loc}(a, b) \leq 1$. Since 1 can be increased without violating the constraints that $x^{loc}(a, b) \leq 1$ and exists, $\tilde{a}$ would not meet the requirement of being the upper bound on the interval of 1's such that these constraints are satisfied. By contrast, if $U(x, a, b)$ has a single critical point, 1 cannot be increased further as the function would then have no critical points. Since $U(x, a, b)$ has a single critical point, it is non-decreasing in x. Thus, $x^{loc}(\tilde{a}, b) < 1$ implies $U\left(x^{loc}(\tilde{a}, b), \tilde{a}, b\right) < U(1, \tilde{a}, b)$. Facts 2 and 3 imply that $\frac{\partial U\left(x^{loc}(a,b),a,b\right)}{\partial a} < \frac{\partial U(1,a,b)}{\partial a}$, and we also have $U\left(x^{loc}(0, b), 0, b\right) < U(1, 0, b)$. Thus, the functions $U\left(x^{loc}(a, b), a, b\right)$ and $U(1, a, b)$ must cross at some point $a^*(b) \in [0, a]$. Finally, Fact 4 implies that on $[0, a^*(b))$, $x^*(a, b) = x^{loc}(a, b)$. ■

Lemma 2 *(i) If* $b > \frac{1}{2}$ *then* $x^*(a, b)$ *is strictly increasing on* $[0, a^*(b))$.

(ii) If $b \in (\frac{1}{2}, 1]$ *then* $\frac{b - \sqrt{b^2 - 3a^*(b)}}{3a^*(b)} = 1$ *and* $x^*(a, b)$ *smoothly increases up to 1.*

(iii) If $b > 1$ *then* $\frac{b - \sqrt{b^2 - 3a^*(b)}}{3a^*(b)} < 1$ *and* $x^*(a, b)$ ***explodes*** *up to 1 upon reaching the critical threshold* $a^*(b)$.

Proof. The first claim follows from Fact 1 in the proof of Lemma 1. For the third claim, note $x^{loc}(a,b)$ is only defined when $a \leq \frac{b^2}{3}$ and $x^{loc}(\frac{b^2}{3},b) = \frac{1}{b}$. Fact 1 then implies the $b > 1$ claim. For the second claim, now suppose $b \leq 1$. Then $x^{loc}(\frac{b^2}{3},b) = \frac{1}{b} \geq 1$ and it is also the inflection point. In general the inflection point is $\frac{b}{3a}$. Thus as 1 decreases from $\frac{b^2}{3}$, the inflection point is increasing. In particular, it remains above 1. However, the only way that we can have $U\left(1, a^*\left(b\right), b\right) > U\left(1, x^{loc}\left(a^*\left(b\right)\right), b\right)$ (i.e. an explosion) is if both $x^{loc}(a^*(b),b)$ and the inflection point are both strictly smaller than 1. Thus, there is no explosion. ■

Lemma 3 *If $b > \frac{1}{2}$ then the quantities $bx^*(a,b) - ax^{*2}(a,b)$ and $x^*(a,b)(bx^*(a,b) - ax^{*2}(a,b))$ are both increasing on $[0, a^*(b))$.*

Proof. On $[0, a^*(b))$

$$\frac{\partial}{\partial x}U(x,a,b)|_{x^*(a,b)} = 1 - 2bx^*(a,b) + 3ax^{*2}(a,b) = 0$$

$$\Rightarrow \frac{\partial}{\partial a}U(x^*(a,b),a,b) = -2bx_1^*(a,b) + 6ax^*(a,b)x_1^*(a,b) + 3x^{*2}(a,b) = 0 \tag{31}$$

Now

$$\frac{\partial}{\partial a}bx^*(a,b) - ax^{*2}(a,b) = bx_1^*(a,b) - 2ax^*(a,b)x_1^*(a,b) - x^{*2}(a,b)$$

Equation (31) then implies

$$\frac{\partial}{\partial a}bx^*(a,b) - ax^{*2}(a,b) = \frac{b}{3}x_1^*(a,b) > 0$$

This shows $bx^*(a,b) - ax^{*2}(a,b)$ is increasing. Since $x^*(a,b)$ is positive and increasing as well, so $x^*(a,b)(bx^*(a,b) - ax^{*2}(a,b))$ is also increasing. ■

Proof of Proposition 1

The principal's objective function is $\frac{p_1+p_2}{2}\left(1 - (p_1+p_2) + p_1p_2s\right)$. We first wish to prove that $p_1 = p_2$. Fix a given $X = p_1 + p_2$. The term p_1p_2s is maximized, for a given X, by setting $p_1 = p_2$. The other terms in the objective function are all terms in X. Thus, we have $p_1 = p_2 = p$. This allows us to apply Lemmas 1, 2 and 3 with $x = \frac{p_1+p_2}{2} = p$; statements (i), (ii) and (iii) are essentially transcriptions of these three Lemmas, respectively. The only difference is that at the critical synergy level, we now discriminate between the two optimal efforts in accordance with Assumption 1.

To see (iv), note if i is more influential than j then $h_{ij} > h_{ji}$. This implies:

$$w_i^*(s) = p^*(s)(1 - h_{ji}p^*(s)) > p^*(s)(1 - h_{ij}p^*(s)) = w_j^*(s).$$

More generally, holding synergy fixed, an increase in agent i's relative influence means both increasing h_{ij} and decreasing h_{ji}. This causes both an increase in w_i^* and a decrease in w_j^*, which proves (v).

The proof of part (vi) is as follows. We use a dot to denote the derivative with respect to h_{ij}.

$$\dot{w_j^*} = \dot{p^*} - 2h_{ij}p^*\dot{p^*} - p^{*2}$$

$$p^* \in \arg\max_p p(1 - 2p + s^*p^2) \Rightarrow 1 - 4p^* + 3sp^{*2} = 0 \Rightarrow -4\dot{p^*} + 6sp^*\dot{p^*} + 3p^{*2} = 0$$

A linear combination of the two gives us

$$\dot{w_j^*} = \frac{1}{3}\dot{p^*}\left(6h_{ij}p^* - 1\right)$$

Since $\dot{p^*} > 0$, this means that, when $s < \bar{s}$, $\dot{w_j^*}$ and $6h_{ij}p^* - 1$ have the same sign. Equation (19) follows immediately. Turning to the expected wage, we have:

$$\dot{p^* w_j^*} = 2p^*\dot{p^*} - 3h_{ij}p^{*2}\dot{p^*} - p^{*3}$$

$$-4\dot{p^*} + 6sp^*\dot{p^*} + 3p^{*2} = 0 \Rightarrow p^*\left(-4\dot{p^*} + 6sp^*\dot{p^*} + 3p^{*2}\right) = 0$$

A linear combination of the two gives us

$$\dot{p^* w_j^*} = 3h_{ij}p^{*2}\dot{p^*} + \frac{1}{2}p^{*3} > 0.$$

Finally, for part (vii), the first-order condition yields: $w_1 = p_1\left(1 - h_{21}p_2\right)$. Thus, agent 1's utility is given by:

$$\begin{aligned} U_1 &= p_1\left(1 - h_{21}p_2\right)\left(\frac{p_1 + p_2}{2}\right) - \frac{1}{4}p_1^2\left(1 - h_{21}p_2\right) \\ &= \frac{3}{4}p^2\left(1 - h_{21}p\right) \end{aligned}$$

where $p = p_1 + p_2$, and similarly $U_2 = \frac{3}{4}p^2\left(1 - h_{12}p\right)$. Hence $U_1 > U_2$ if and only if $h_{12} > h_{21}$.

Proof of Proposition 2

Holding total effort constant,

$$p_1^*(\mathbf{s}), p_2^*(\mathbf{s}), p_3^*(\mathbf{s}) \in \arg \max_{p_1,p_2,p_3 \in [0,1]} Ap_1p_2 + Bp_1p_3 + Cp_2p_3 \tag{32}$$

The first-order conditions which characterize interior solutions to this convex problem are captured by equation (23). This proves (i).

Since the maximization problem of equation (32) is convex, the optimal effort profile will satisfy the ratios of equation (24) so long as:

1. Each synergy component is strictly smaller than the sum of the other two.

2. The restriction of each effort being no greater than 1 is nonbinding.

Condition 1 is assumed in this lemma and condition 2 holds if synergy is sufficiently small. Suppose then that synergy is small. Call by p the highest effort of the optimal effort profile. Then there exists $1 \geq \alpha \geq \beta > 0$ such that the other two efforts are αp and βp. Assume without loss of generality that agent 1's effort is highest, agent 2's effort is α times agent 1's effort and agent 3's effort is β times agent 1's effort. Then the principal's maximization problem becomes

$$p^* \in \arg \max_{p \in [0,1]} (1+\alpha+\beta)p\left(1-(1+\alpha+\beta)\,p+(A\alpha+B\beta+C\alpha\beta)\,p^2\right).$$

Statement (ii) now follows from Lemma 1. Statement (iii) follows from Lemma 3.

Holding the synergy profile fixed, an increase in agent i's relative influence means both an increase of at least one element of $\{h_{ij}\}_{j\neq i}$ and a corresponding decrease of some elements in $\{h_{ji}\}_{j\neq i}$. This causes an increase in w_i^* and a decrease in at least one element of $\{w_j^*\}_{j\neq i}$ provided the effort profile is interior. Moreover, since (p_1, p_2, p_3) is a function of the synergy profile only, it is unaffected by changes in relative influence and so p^* is unchanged. Statement (iv) now follows.

Proof of Proposition 3

Without loss of generality, suppose $A > B \geq C$ and $A \geq B + C$. Looking at the convex problem of equation (32), it is clear that $p_3^* = 0$. But then the principal's maximization problem becomes symmetric in p_1 and p_2 and there is nontrivial synergy between agents 1 and 2. The statement in the proposition then follows from the preliminary two-agent case.

Proof of Corollary 1

Recall the optimal wage for agent i is

$$w_i^*(p_i^*) = p_i^* \left(1 - \sum_{j \neq i} h_{ji} p_j^*\right).$$

Equation (23) and the corollary's assumption about the influence parameters imply that the quantity inside the parentheses is the same for all i. The result now follows immediately.

Proof of Proposition 5

The principal solves:

$$\max_{p_1, p_2, w_1 \geq 0, w_2 \geq 0} \left(a\frac{p_1 + p_2}{2} + b\sqrt{p_1 p_2}\right)(1 - w_1 - w_2)$$

subject to

$$p_i \in \arg\max_{p \in [0,1]} \left(a\frac{p + p_{-i}}{2} + b\sqrt{p p_{-i}}\right) w_i - \frac{1}{4}p^2, \quad i = 1, 2.$$

We prove the Proposition using a series of lemmas.

Lemma 4 *Given wages w_1 and w_2, the agents' effort levels satisfy*

$$p_i = \begin{cases} 0 & \text{if } w_i = 0, \\ \left(a + b\sqrt{\frac{p_{-i}}{p_i}}\right) w_i & \text{if } w_i > 0 \text{ and } (a + b\sqrt{p_{-i}})w_i < 1, \\ 1 & \text{if } (a + b\sqrt{p_{-i}})w_i \geq 1, \end{cases}$$

for $i = 1, 2$.

Proof. Let

$$f_i(p) = \left(a\frac{p + p_{-i}}{2} + b\sqrt{p p_{-i}}\right) w_i - \frac{1}{4}p^2, \quad i = 1, 2,$$

denote agent i's expected utility function. Then, the first and second order derivatives are

$$f_i'(p) = \left(\frac{a}{2} + \frac{b}{2}\sqrt{\frac{p_{-i}}{p}}\right) w_1 - \frac{1}{2}p, \quad i = 1, 2,$$

$$f_i''(p) = -\frac{1}{4}\left(\frac{b}{p}\sqrt{\frac{p_{-i}}{p}} w_1\right) - \frac{1}{2} < 0, \quad i = 1, 2.$$

Since f_i is strictly concave and $[0, 1]$ is a compact set, there is a unique maximizer p. Moreover, if $f_i'(0) \leq 0$, then $p = 0$. If $f_i'(1) \geq 0$, then $p = 1$. Otherwise, if

$f_i'(0) > 0 > f_i'(1)$, then there is a unique $p \in (0,1)$ given by the first-order condition. Specifically, note that $f_i'(0) = 0$ if $w_i = 0$, and for $w_i > 0$,

$$0 \geq f_i'(0) = \left(\frac{a}{2} + \frac{b}{2}\lim_{p \downarrow 0}\sqrt{\frac{p_{-i}}{p}}\right) w_i = \begin{cases} \frac{1}{2}aw_i & \text{if } p_{-i} = 0, \\ +\infty & \text{if } p_{-i} > 0, \end{cases}$$

so $p = 0$ if $w_i = 0$;

$$0 \leq f_i'(1) = \left(\frac{a}{2} + \frac{b}{2}\sqrt{p_{-i}}\right) w_i - \frac{1}{2},$$

so $p = 1$ if $w_i \geq \frac{1}{a+b\sqrt{p_{-i}}}$; and, finally, in all other cases, the first-order condition implies that $p \in (0,1)$ satisfies

$$p = \left(a + b\sqrt{\frac{p_{-i}}{p}}\right) w_i.$$

■

We will now substitute the wages into the principal's problem, so that it becomes a function of the effort levels p_i only.

Lemma 5 *The principal's problem reduces to*

$$(p_i^*, p_2^*) \in \arg\max_{\substack{0 \leq p_1 \leq 1, \\ 0 \leq p_2 \leq 1}} \left(a\frac{p_1 + p_2}{2} + b\sqrt{p_1 p_2}\right)\left[1 - \left(\frac{p_1}{a + b\sqrt{\frac{p_2}{p_1}}} + \frac{p_2}{a + b\sqrt{\frac{p_1}{p_2}}}\right)\right].$$

with optimal success wages given by

$$w_i^* = \begin{cases} 0 & p_i^* = 0, \\ \frac{p_i^*}{a+b\sqrt{\frac{p_{-i}^*}{p_i^*}}}, & 0 < p_i^* \leq 1. \end{cases} \tag{33}$$

Proof. Let $W_i^p = \{w_i \geq 0 : p_i = p\}$. By Lemma 4, $W_i^1 = [\frac{1}{(a+b\sqrt{p_{-i}})}, \infty)$. Since $\Pr(r = 1)$ is a nonnegative constant for all $w_i \in W_i^1$, and $1 - w_1 - w_2$ is decreasing in w_i, it is enough to limit consideration to the smallest element in W_i^1, which is $\frac{p_i}{(a+b\sqrt{p_{-i}})}$ evaluated at $p_i = 1$. By Lemma 4, $W_i^0 = \{0\}$, so $w_i = 0$ for $p_i = 0$, and $w_i = \frac{p_i}{(a+b\sqrt{\frac{p_{-i}}{p_i}})}$ for each $p \in (0,1)$. Hence, we can replace w_i in the objective function with $w_i = \frac{p_i}{(a+b\sqrt{\frac{p_{-i}}{p_i}})}$, where we take $w_i = 0$ for $p_i = 0$ since $\lim_{p_i \downarrow 0} \frac{p_i}{(a+b\sqrt{\frac{p_{-i}}{p_i}})} = 0$. ■

Equation (33) is part (ii) of Proposition 5. It now remains to prove parts (i) and (iii). To solve the problem of Lemma 5, we introduce the following functions. Let

$$f(p,q) = g(p,q)(1 - h(p,q)), \tag{34}$$

where the functions C^1 and C^2 are defined by

$$g(p,q) = a\frac{p+q}{2} + b\sqrt{pq}, \tag{35}$$

$$h(p,q) = \frac{p}{a + b\sqrt{\frac{q}{p}}} + \frac{q}{a + b\sqrt{\frac{p}{q}}}, \tag{36}$$

with $h(0,z) = h(z,0) = \frac{z}{a}$.

Then, the reduced problem is equivalent to solving

$$\max_{\substack{0 \le p_1 \le 1, \\ 0 \le p_2 \le 1}} f(p_1, p_2). \tag{37}$$

Remark 1 *Note that this problem* (37) *is not concave. For example, let* $a = \frac{1}{10}$, $b = \frac{9}{10}$, *and consider* $p \in \{\frac{1}{500}, \frac{1}{50}, \frac{1}{5}\}$ *for* $q = \frac{9}{10}$. *Then,* $f(\frac{1}{500}, \frac{9}{10}) \approx -0.443$, $f(\frac{1}{50}, \frac{9}{10}) \approx -0.475$, *and* $f(\frac{1}{5}, \frac{9}{10}) \approx -0.357$. *So,* $\frac{1}{500} < \frac{1}{50} < \frac{1}{5}$, *but* $f(\frac{1}{500}, \frac{9}{10}) > f(\frac{1}{50}, \frac{9}{10}) < f(\frac{1}{5}, \frac{9}{10})$, *which violates concavity. Nevertheless, this problem does have a unique global maximum, which we demonstrate below.*

The next few lemmas help establish that the solution involves symmetric effort levels, $p_1 = p_2$.

Lemma 6 *For all* $p, q \ge 0$, *there exist* $x, y \ge 0$ *such that the functions* C^1 *and* C^2 *in* (35) *and* (36) *satisfy*

$$g(p,q) = g(x,x) \quad \text{and} \quad h(p,q) = h(y,y).$$

Namely,

$$x = \frac{a}{a+b}\frac{p+q}{2} + \frac{b}{a+b}\sqrt{pq}, \tag{38}$$

$$y = \frac{a+b}{2}\left(\frac{p}{a + b\sqrt{\frac{q}{p}}} + \frac{q}{a + b\sqrt{\frac{p}{q}}}\right), \tag{39}$$

$$g(x,x) = (a+b)x, \tag{40}$$

and

$$h(y,y) = \frac{2y}{a+b}. \tag{41}$$

Proof. The result follows by substitution of (38) into (35) and (39) into (36). ■

Lemma 7 *The functions C^1 and C^2 are homogeneous of degree one. Namely, for all $p, q \geq 0$, and for all $\alpha \geq 0$, we have $g(\alpha p, \alpha q) = \alpha g(p,q)$ and $h(\alpha p, \alpha q) = \alpha h(p,q)$.*

Proof. The result follows immediately from (35) and (36). ■

Lemma 8 *For all $p, q \geq 0$, the values x and y defined in Lemma 6 satisfy $x \leq y$. Moreover, $x < y$ if $p \neq q$.*

Proof. Let $\beta = \frac{b}{a+b}$. Then, $x \leq y$ if and only if

$$(1-\beta)\frac{p+q}{2} + \beta\sqrt{pq} \leq \frac{1}{2}\left(\frac{p}{(1-\beta)+\beta\sqrt{\frac{q}{p}}} + \frac{q}{(1-\beta)+\beta\sqrt{\frac{p}{q}}}\right). \tag{42}$$

Note that both sides of (42) are zero for $p = q = 0$. Moreover, if $p = 0$, then condition (42) reduces to

$$\frac{1}{2}(1-\beta)q \leq \frac{1}{2}\frac{1}{1-\beta}q,$$

which always holds since $0 < 1-\beta < 1 < \frac{1}{1-\beta}$. A similar property holds for p when $q = 0$, so $x \leq y$ when either $p = 0$ or $q = 0$.

Now, suppose that $p, q > 0$. Multiplying both sides of (42) by $\frac{2}{\sqrt{pq}}$ yields the equivalent condition

$$(1-\beta)\left(\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}\right) + 2\beta \leq \frac{\sqrt{\frac{p}{q}}}{(1-\beta)+\beta\sqrt{\frac{q}{p}}} + \frac{\sqrt{\frac{q}{p}}}{(1-\beta)+\beta\sqrt{\frac{p}{q}}} \tag{43}$$

Let

$$w = \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}.$$

Note that $w \geq 2$ for all $p, q > 0$. This lower bound on w follows by substitution of $\alpha = \sqrt{\frac{p}{q}}$ and minimization:

$$\min_{p,q>0} \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = \min_{\alpha>0} \alpha + \frac{1}{\alpha} = 2,$$

which follows because $\alpha + \frac{1}{\alpha}$ is a strictly convex function on $\alpha > 0$ that is minimized at $\alpha = 1$ on $\alpha > 0$.

Now by substitution and simplification of the right-hand side of (43) into a single ratio, we have $x \leq y$ if and only if

$$(1-\beta)w + 2\beta \leq w\frac{(1-\beta) - 2\beta\frac{1}{w} + \beta w}{(1-\beta)^2 + \beta(1-\beta)w + \beta^2}. \tag{44}$$

Note that the left-hand side of (44) is a convex-combination of 2 and w, and because $w \geq 2$, is at most w. The right-hand side of (44) is at least w. To see this, note that because $w \geq 2$ and $\beta > 0$,

$$1 + \beta(w-2) \geq 1 \geq \frac{2}{w},$$

which after multiplying by β and rearranging gives

$$\beta^2 w + \beta(1-2\beta) \geq \frac{2\beta}{w}.$$

Now, using the identities $\beta^2 = \beta - \beta(1-\beta)$ and $\beta(1-2\beta) = (1-\beta) - [(1-\beta)^2 + \beta^2]$, we have

$$[\beta - \beta(1-\beta)]w + (1-\beta) - [(1-\beta)^2 + \beta^2] \geq \frac{2\beta}{w},$$

which can be rearranged as

$$(1-\beta) - 2\beta\frac{1}{w} + \beta w \geq (1-\beta)^2 + \beta(1-\beta)w + \beta^2.$$

This inequality implies that

$$\frac{(1-\beta) - 2\beta\frac{1}{w} + \beta w}{(1-\beta)^2 + \beta(1-\beta)w + \beta^2} \geq 1;$$

hence, the term multiplying w on the right-hand side of (44) is at least one. Thus, the right-hand side of (44) is at least w and the left-hand side of (44) is at most w. Therefore, $x \leq y$ for all $p, q > 0$. Furthermore, all of the above inequalities are strict if $p \neq q$ since $w > 2$, which leads to $x < y$. ■

Lemma 9 *For all $p, q \geq 0$, the function f in* (34) *satisfies*

$$f(p,q) \leq \frac{(a+b)^2}{8}.$$

Moreover, the inequality is strict for $p \neq q$.

Proof. First, note that $f(0,0) = 0$, and that for all $z \geq 0$,

$$f(0,z) = f(z,0) = \frac{1}{2}az\left(1 - \frac{z}{a}\right),$$

which is strictly concave in z on $z \geq 0$. Hence, the first-order condition yields the optimal $z = \frac{a}{2}$ and

$$f(0,z) = f(z,0) \leq f\left(\frac{a}{2}, 0\right) = \frac{a^2}{8} < \frac{(a+b)^2}{8}.$$

So, the result holds if $p = 0$ or $q = 0$. Now, suppose $p, q > 0$. The maximum value of f along the ray $\{(\alpha p, \alpha q) : \alpha \geq 0\}$ is given by

$$\max_{\alpha \geq 0} f(\alpha p, \alpha q) = \max_{\alpha \geq 0} \alpha g(p,q)(1 - \alpha h(p,q)) = \frac{1}{4}\frac{g(p,q)}{h(p,q)}.$$

The first equality follows by application of Lemma 7. The second equality follows from the first-order condition, which gives $\alpha = \frac{1}{2h(p,q)} > 0$ as the unique maximizer since the objective is strictly concave on $\alpha \geq 0$.

By Lemma 6, there exist $x, y \geq 0$ such that

$$\max_{\alpha \geq 0} f(\alpha p, \alpha q) = \frac{1}{4}\frac{g(p,q)}{h(p,q)} = \frac{1}{4}\frac{g(x,x)}{h(y,y)} = \frac{(a+b)^2}{8}\frac{x}{y}.$$

By Lemma 8, we have $\frac{x}{y} \leq 1$, with the inequality strict for $p \neq q$. Thus,

$$f(p,q) \leq \frac{(a+b)^2}{8},$$

with the inequality strict for $p \neq q$. ■

Lemma 10 *The solution to the model in which the synergy is in the production function, not the cost function, is the following. The agents exert the same level of effort*

$$p_1^* = p_2^* = \frac{a+b}{4},$$

and receive the same success wage

$$w_1^* = w_2^* = \frac{1}{4},$$

which results in optimal expected utility $\frac{a+b}{16}(1-\frac{a+b}{4})$ *for each agent and optimal profit* $\frac{(a+b)^2}{8}$ *for the principal.*

Proof. The reduced problem of (37) when $p_1 = p_2 = p$ is

$$\max_{p\geq 0} f(p,p) = \max_{p\geq 0} g(p,p)(1-h(p,p)) = \max_{p\geq 0}(a+b)p\left(1-\frac{2p}{a+b}\right) = \frac{(a+b)^2}{8}.$$

The second equality follows by Lemma 6. The third equality follows by the first order condition of the function being maximized, which is strictly concave in p on $p \geq 0$ and has the unique maximizer $p = \frac{a+b}{4}$. Hence, $p_1 = p_2 = \frac{a+b}{4}$ is the unique maximizer among all symmetric effort levels, and by Lemma 9 it is the unique global maximizer since it achieves an objective value that is strictly larger than that achieved by any non-symmetric effort levels. The optimal success wages are obtained by (33). ■

Lemma 10 proves parts (i) and (iii), and so the Proposition is proven.

B Complementary Effort

In Sections 3 and 4, efforts were perfect substitutes in the production function. This section considers a model in which agents' efforts are perfect complements, i.e. the probability of success depends on the minimum effort level undertaken by all agents, and shows that the results are robust. The production function now becomes*:*

$$\Pr(r=1) = \min\left(p_1, p_2, ..., p_N\right). \tag{45}$$

We continue to assume a quadratic individual cost function:

$$h_i(p_i) = \frac{\kappa_i}{2}p_i^2.$$

Differentiating agent i's utility function (5) gives his first-order conditions as:

$$p_1 = p_2 = \ldots = p_N \equiv p, \tag{46}$$

and

$$w_i(p) = \kappa_i p\left(1 - \sum_{j\neq i} h_{ji}p\right). \tag{47}$$

These first-order conditions already give us some preliminary results. Equation (46) shows that all agents will exert the same effort level, as is intuitive given the perfect complementarities production function (45). Equation (47) shows that agent i's wage is linear in his cost parameter κ_i, i.e. agents with more difficult tasks (higher κ_i) will receive higher wages.

Plugging the first-order conditions (46) and (47) into the principal's objective function (3) gives her reduced-form maximization problem as:

$$p^* \in \arg\max_p \; p\left(1 - \sum_i w_i(p)\right) = \arg\max_p \; p\left(1 - p\sum_i \kappa_i + p^2 \sum_i \left(\sum_{j \neq i} h_{ij}\kappa_j\right)\right).$$

We define the following terms:

Definition 3 ***Synergy*** *is defined to be the sum of each agent's total influence:*

$$s = \sum_i \left(\sum_{j \neq i} h_{ij}\kappa_j\right)$$

Difficulty *is defined to be the sum of the cost parameters,* $\kappa \equiv \sum_i \kappa_i$.

Assumption 2 *Difficulty* $\kappa > \frac{1}{2}$.

This is a nontriviality assumption about the difficulty of the project being not too low. It ensures that the problem has nontrivial solutions in agent efforts for at least some realized levels of synergy.

The solution to the model is given by Proposition 6 below; the proof is essentially the same as in Proposition 1.

Proposition 6 *(Complementary production function.) (i) There exists a unique critical synergy threshold* $s^*(\kappa) > 0$ *such that optimal effort is given by:*

$$p^*(s) = \begin{cases} \frac{\kappa - \sqrt{\kappa - 3s}}{3s} & s \in [0, s^*(\kappa)) \\ 1 & s \geq s^*(\kappa). \end{cases}$$

Optimal effort $p^*(s)$ *is strictly increasing on* $[0, s^*(\kappa)]$. *Furthermore, if difficulty* $\kappa > 1$, *then* $p^*(s)$ *explodes to 1 when the critical synergy level* $s^*(\kappa)$ *is reached.*

(ii) Total wages given success, $w^*(s) = \sum_i w_i^*(s)$, *and expected total wages* $p^*(s)w^*(s)$ *are both strictly increasing on* $[0, s^*(\kappa)]$.

(iii) Suppose synergy is subcritical. An increase in any influence parameter of any agent will lead to increases in optimal effort, total payment given success and total expected success payment.

(iv) Fix a subcritical synergy level. Suppose agent i's relative influence increases, i.e. his total influence increases while holding synergy constant. If the resulting decrease in the total influence of the other agents is nondistortionary[10] *then there is an increase in agent i's relative and absolute wage. Specifically,*

$$\frac{w_i^*}{\sum_j w_j^*},\ w_i^* \ \textit{and} \ \ p^* w_i^* \ \ \textit{all strictly increase,}$$

and

$$\frac{w_i^*}{w_j^*} \ \ \textit{weakly increases for all } j \textit{ and strictly increases at least one } j.$$

Proposition 6 shows that our model's key results are robust to the nature of the production function. Even though the perfect complements production function of this section is the polar opposite of the perfect substitutes production function of Sections 3 and 4, the main insights regarding the effort and wage profiles remain unchanged. In addition to demonstrating robustness to the specification of the production function, this section also shows that the results naturally extend to the case of N agents.

As in Sections 3 and 4, an increase in total synergy leads to an increase in the implemented effort levels, total pay and expected total pay; the intuition is the same. An increase in a single agent's influence parameters augments total synergy (thus leading to the above effects) and his own pay in both relative and absolute terms.

[10] In other words, the decrease in the other agents' total influence is achieved by simply multiplying their influence parameters with a common scalar $c < 1$.

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